All The World’s NP-Complete, And All of Us Merely P...

Every problem in this assignment (except the optional challenge problem) involves transforming some problem $X$ into another problem $Y$ such that

- Any instance of problem $X$ satisfying some property $Q$ corresponds to an instance of problem $Y$ satisfying some (hopefully related!) property $Q'$, and
- Any instance of problem $X$ not satisfying property $Q$ corresponds to an instance of problem $Y$ not satisfying property $Q'$. This is often proven by contrapositive by assuming that the instance of problem $Y$ does satisfy $Q'$ and showing that the instance of problem $X$ from which it was derived must have satisfied property $Q$.
- The instance of problem $Y$ constructed from an instance of problem $X$ is not too big.

Be sure to establish each of these facts in your solutions!

Notes:

- In proving NP-completeness for a problem $Y$, time spent selecting the most appropriate NP-complete problem $X$ to reduce from is time well spent!
- Students often ask “Which problems can I use as the ‘known’ NP-complete problem?” You can use any of the problems whose NP-completeness was established in class, the slides, or Chapter 8 of the text. These are:
  - Independent Set
  - Vertex Cover
  - Set Cover
  - SAT, 3SAT, ATMOST3SAT, CIRCUITSAT
  - Hamiltonian Cycle, Hamiltonian Path, Longest Path, Decision version of Traveling Salesperson Problem
  - Subset Sum, Knapsack
  - 3-D Matching
  - Graph 3-Coloring, Graph $k$-Coloring ($k \geq 3$)
  - Circular Arc $k$-Coloring

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**Question 1.** *Chapter 8, Problem 5: Hitting Set.*

**Question 2.** *Chapter 8, Problem 6: Monotone Satisfiability with Few True Variables.*

**Question 3.** *Chapter 8, Problem 14: Interval Scheduling.*

**Question 4.** *Chapter 8, Problem 17: Zero-Weight Cycles.*

**Hint:** This seems like a sequencing problem, but you might find it easier to reduce from SubSet Sum

**Question 5.** *Chapter 8, Problem 32: Perfect Assembly.*

**Optional Challenge**

**Question 6.** *Chapter 8, Problem 23: Bad Proofs and Pidgeons.*