LECTURE 11

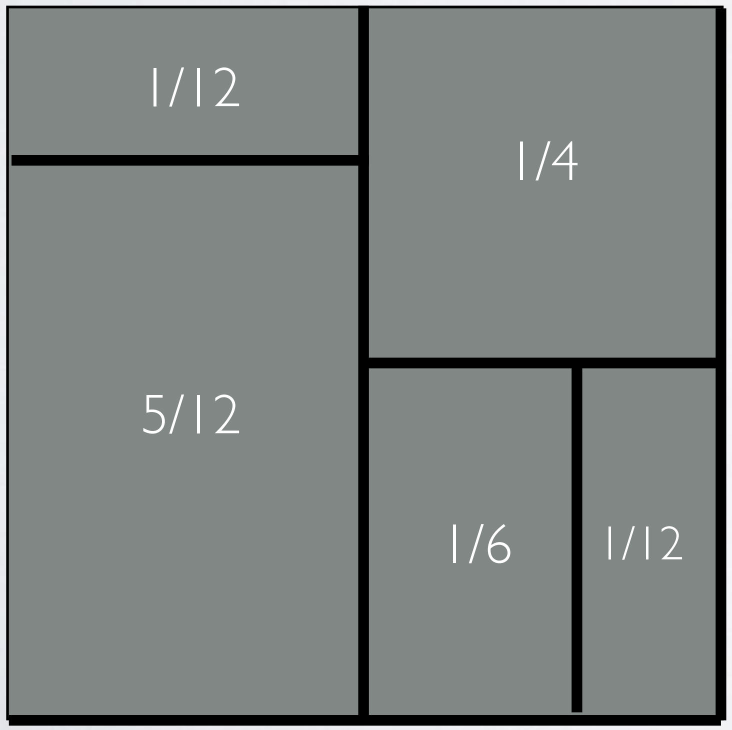
minimum perimeter tilings of the unit square by rectangles

MINIMUM PERIMETER TILING OF THE UNIT SQUARE INTO RECTANGLES

Input: *n* areas $a = a_1, \ldots, a_n$ such that $\sum_{i=1}^{n} a_i = 1$

Output: *n* rectangles $r = r_1, \ldots, r_n$ of matching area that tile the unit square

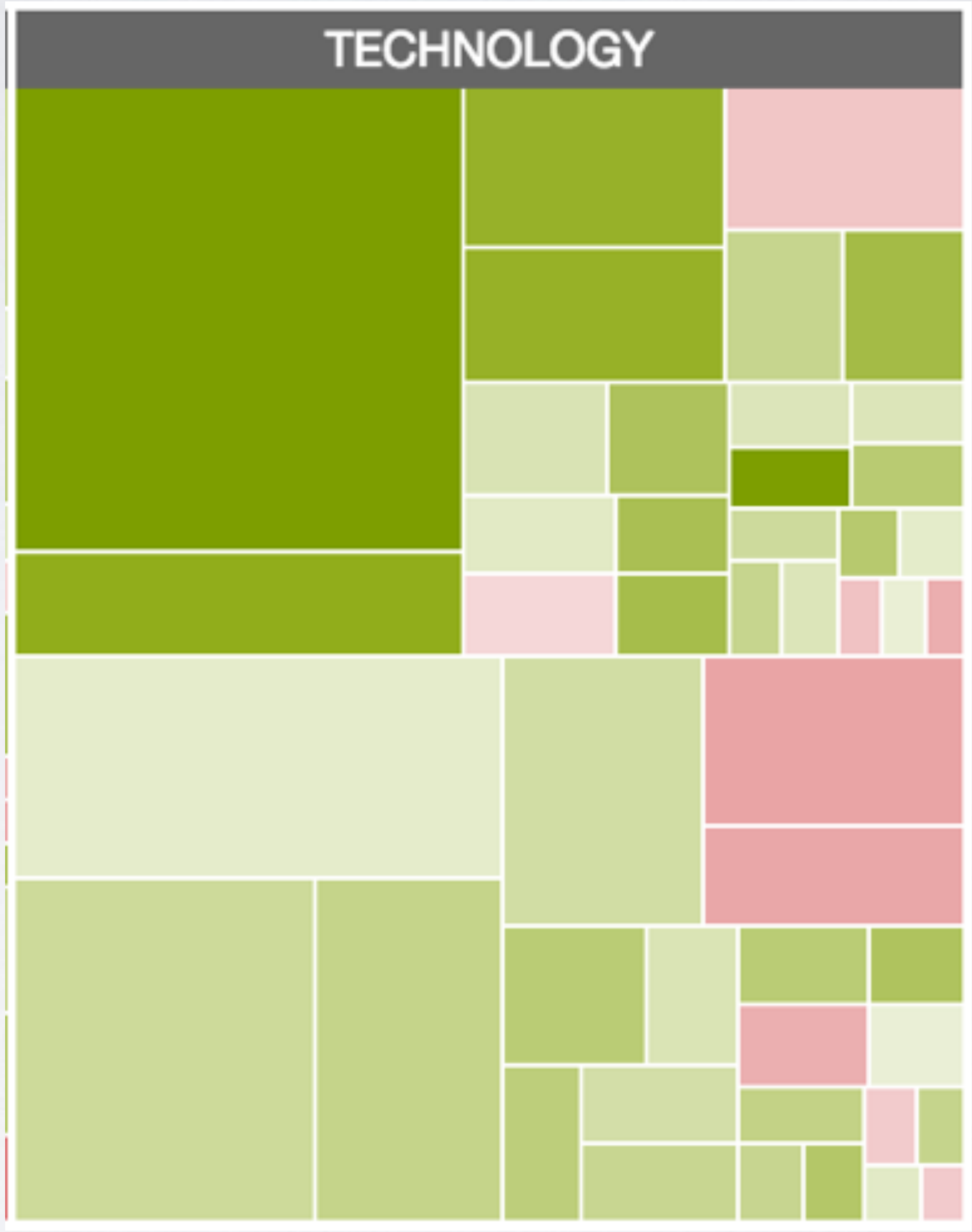
Goal: find the tiling that has minimum total half-perimeter



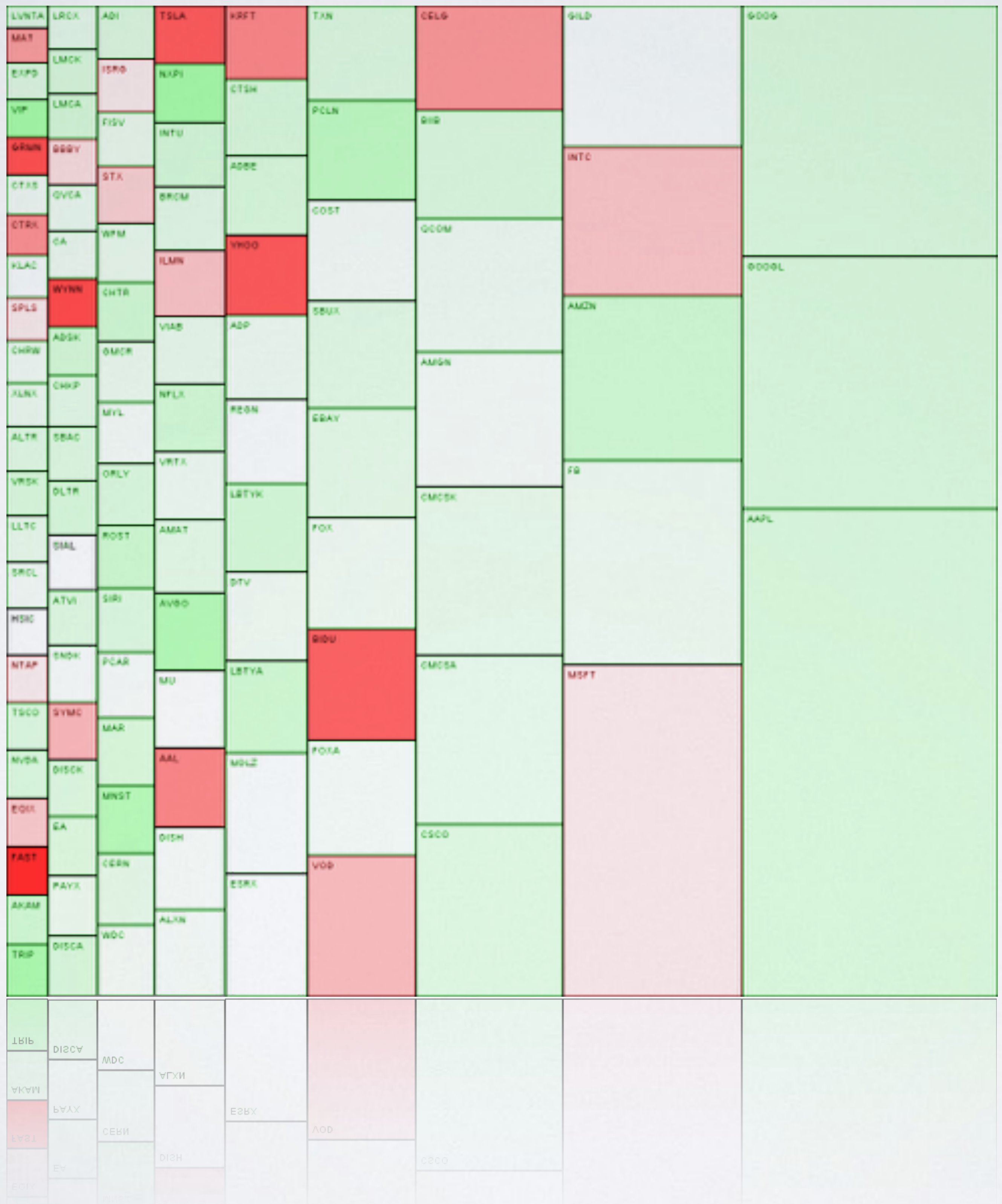
half-perimeter = 1+1/2 +1/2 + 1/2 + 2

= 4 1/2

VISUALIZING MARKET CAP AND CHANGE IN SHARE PRICE



VISUALIZING MARKET CAP AND CHANGE IN SHARE PRICE

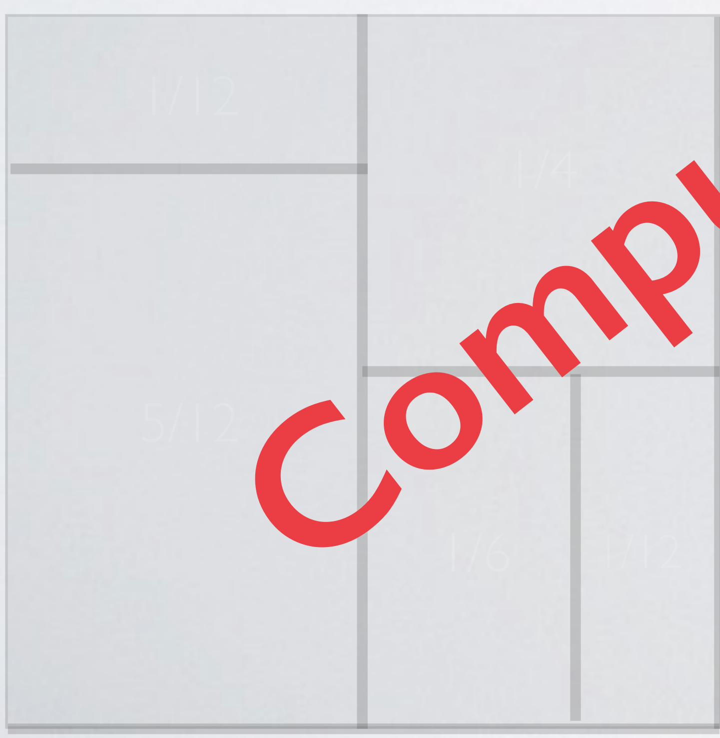


MINIMUM PERIMETER TILING OF THE UNIT SQUARE INTO RECTANGLES

Input: *n* $a = a_1, \ldots, a_n$ such that $\sum_{i=1} a_i = 1$

Output: *n* $r = r_1, \ldots, r_n$ that tile the unit square

Goal: find the tiling that has minimum total half-perimeter

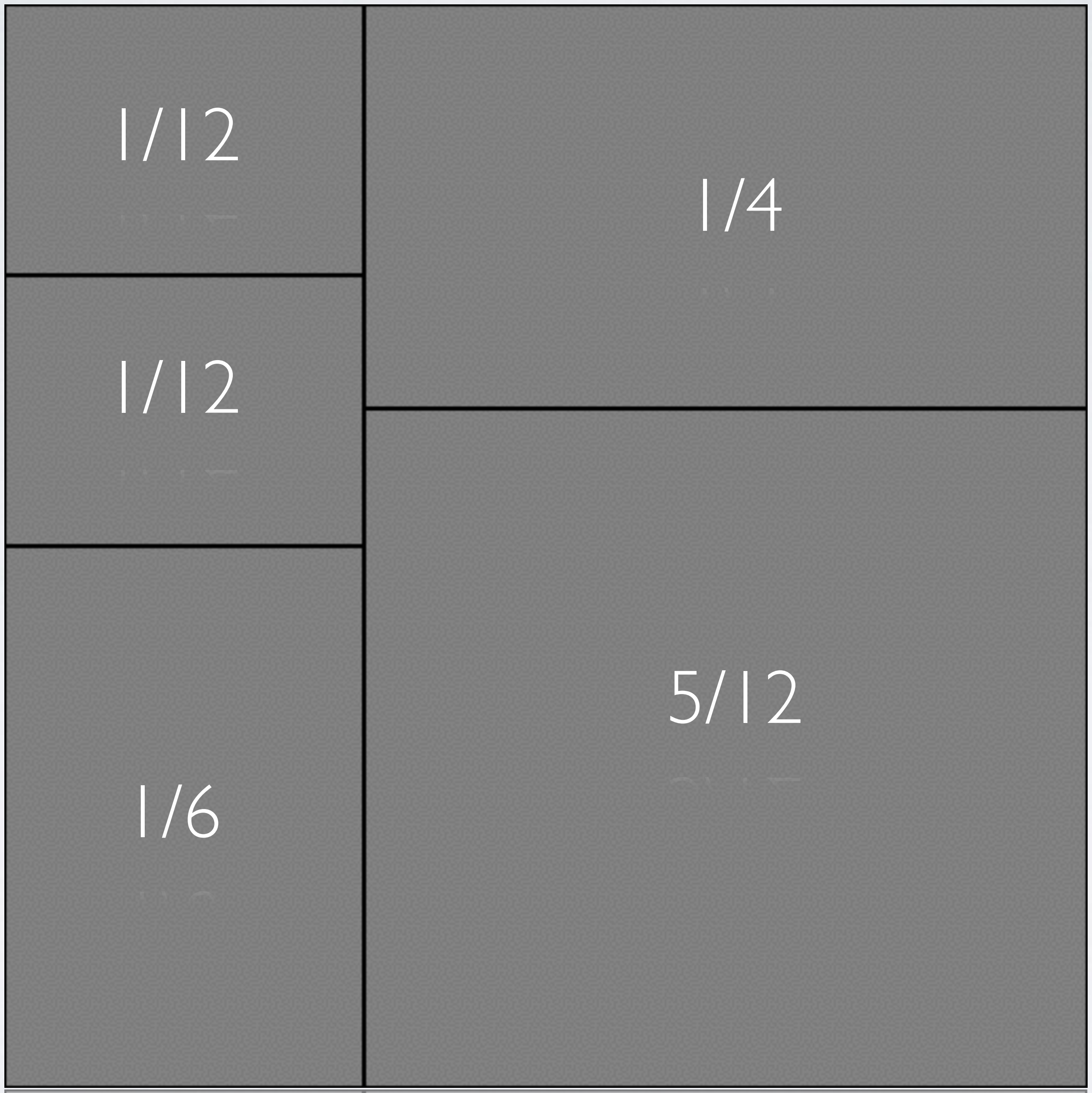


half-perimeter = 1+1/2 +1/2 + 1/2 + 2

= 4 1/2

Computational Intractable!

CONSTRAINT THE PROBLEM: COLUMN-BASED PARTITIONS



COLUMNS

	a_1	a_2	a_3	a_4	a_5
1					
2					
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1					
2					
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08				
2					
3					
4					
5					

$$T[i][j] = \text{half-perimeter of an optimal partitioning of a rectangle with area } \sum_{k=1}^{j} a_k \text{ into } j \text{ rectangles with areas } a_1 \ldots a_j$$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33			
2					
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33	2.0		
2					
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33	2.0	3.33	6.0
2					
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS	a_1 1/12	a_2 1/12	a_3 1/6	a_4 1/4	a_5 5/12
1	1.08	1.33	2.0	3.33	6.0
2		2.16			
3					
4					
5					

$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

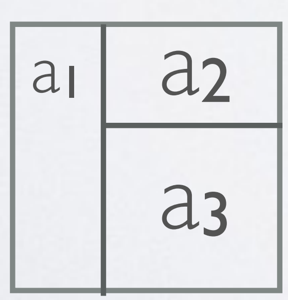
	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33	2.0	3.33	6.0
2		2.16			
3					
4					
5					

COLUMNS

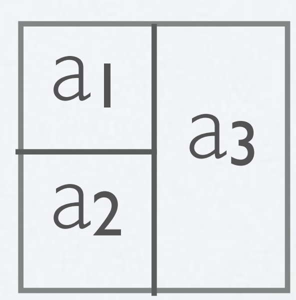
$T[i][j] =$ half-perimeter of an optimal partitioning of a rectangle with area $\sum_{k=1}^{j} a_k$ into j rectangles with areas $a_1 \ldots a_j$

COLUMNS

	1/12	1/12	1/6	1/4	5/12
	a_1	a_2	a_3	a_4	a_5
1	1.08	1.33	2.0	3.33	6.0
2		2.16			
3					
4					
5					



2 7/12



2 1/2

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33	2.0	3.33	6.0
2		2.16			
3					
4					
5					

$$T[i,j] = min_{j'}(1 + (\sum_{j'<k<j} a_k \times (j - j')) + T[i-1, j'])$$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08	1.33	2.0	3.33	6.0
2		2.16	2.5	3.16	4.33
3			3.33	3.75	4.58
4				4.58	5.16
5					6

$$T[i,j] = min_{j'}(1 + (\sum_{j'<k<j} a_k \times (j - j')) + T[i-1, j'])$$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08 [0]	1.33 [0]	2 [0]	3.33 [0]	6 [0]
2		2.16 [1]	2.5 [2]	3.16 [2]	4.33 [3]
3			3.33 [2]	3.75 [3]	4.58 [4]
4				4.58 [3]	5.16 [4]
5					6 [4]

$$T[i,j] = min_{j'}(1 + (\sum_{j'<k<j} a_k \times (j - j')) + T[i-1, j'])$$

COLUMNS	1/12 a_1	1/12 a_2	1/6 a_3	1/4 a_4	5/12 a_5
1	1.08 [0]	1.33 [0]	**2 [0]**	3.33 [0]	6 [0]
2		2.16 [1]	2.5 [2]	3.16 [2]	**4.33 [3]**
3			3.33 [2]	3.75 [3]	4.58 [4]
4				4.58 [3]	5.16 [4]
5					6 [4]

PARTITION = [3]

	1/12 a_0	1/12 a_1	1/6 a_2	1/4 a_3	5/12 a_4
0	1.08	1.33	2	3.33	6
1		2.16 [0]	2.5 [1]	3.16 [1]	4.33 [2]
2			3.33 [1]	3.75 [2]	4.58 [3]
3				4.58 [2]	5.16 [3]
4					6 [3]

COLUMNS

0-based Implementation: PARTITION = [0,3,5]

SLICES = A[0:3], A[3:5]

Given N areas **`AREAS`** and a partition **`PART`** produce N rectangles corresponding to the partition of the areas