CS134 Lecture 20:

More Recursion

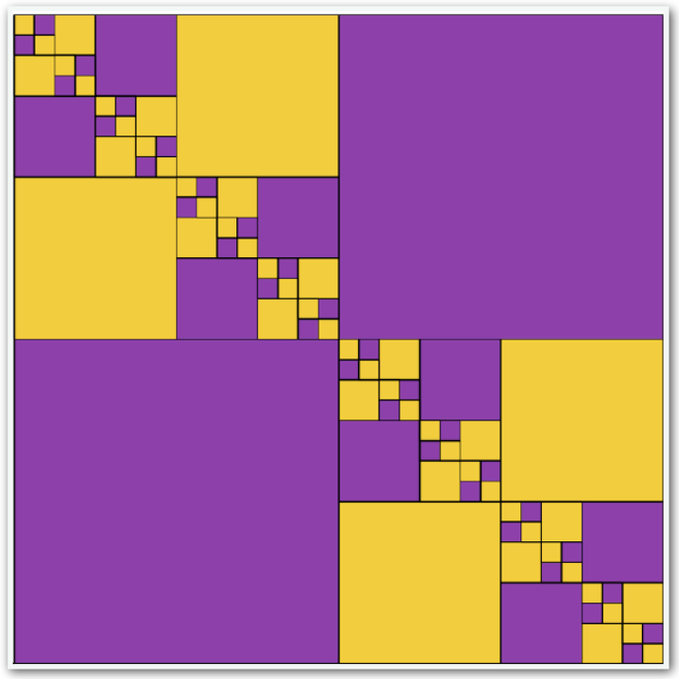
Announcements & Logistics

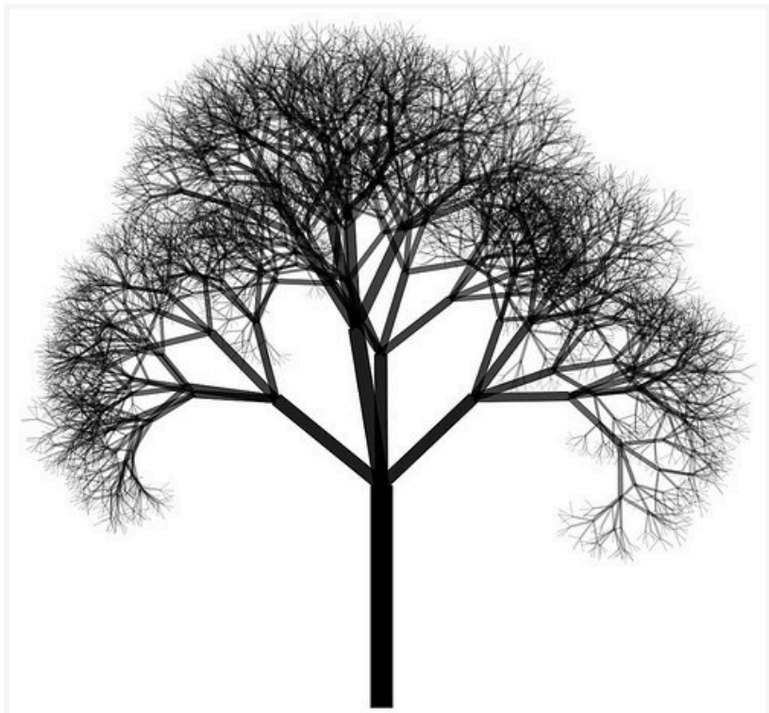
- **HW 6** on GLOW due Mon at 10pm
  - Good practice for short-code questions on exam
  - Practice on pencil and paper first
- Lab 7, 8, and 9 are **partner labs**
  - Pair programming is an important skill as well as a vehicle for learning
- Colloquium Today: **Tim Randolph '18**
  - Theoretical computer science talk on the Subset Sum problem (a problem you may use a "brute-force" approach to solve recursively in a future assignment!)

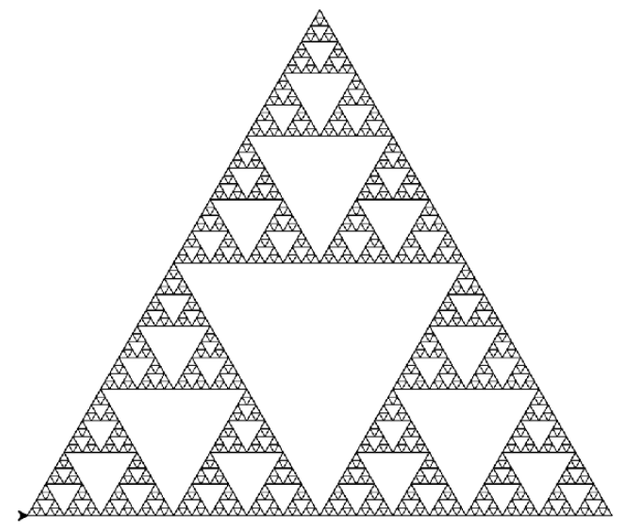
Do You Have Any Questions?

Last Time

- Introduction to recursion
  - Alternative to iteration
  - New problem solving paradigm
- Function frame model to understand recursion behind the scenes







Last Time: Recursive Approach to Problem Solving

- A recursive function is a function **that calls itself**
- A recursive approach to problem solving has two main parts:
  - **Base case(s).** When the problem is **so small**, we solve it directly, without having to reduce it any further
  - **Recursive step.** Does the following things:
    - Performs an action that contributes to the solution
    - **Reduces** the problem to a smaller version of the same problem, and calls the function on this **smaller subproblem**
- The recursive step is a form of "wishful thinking" (also called the inductive hypothesis)

More Recursion:

`count_up`

count_up(n)

- Write a recursive function that prints integers from `1` up to `n` and then prints `"DONE!"`
- Recursive definition of count_up:
  - Base case: `n <= 0, pass # print nothign`
  - Recursive rule: `call count_up(n-1), print(n)`

```
>>> count_up(5)
1
2
3
4
5
```

```
>>> count_up(4)

1
2
3
4
```

```
>>> count_up(3)

1
2
3
```

`count_up(n)`

- Unlike `count_down(n)` the print statement is **after** the recursive function call (*why*?)
- By printing **after** the recursive call, the print statement gets executed "on the way back" from recursive calls

```
def count_up(n):
    '''Prints out integers from 1 up to n'''
    if n <= 0:
        pass  # can omit this if of course
    else:
        count_up(n-1)
        print(n)
```

```
>>> count_up(5)
1
2
3
4
5
```

Function Frame Model to Understand `count_up`

count_up(4)

n 4

```
if n <= 0:
    pass
else:
    count_up(n-1)
    print(n)
```

count_up(3)

n 3

```
if n <= 0:
    pass
else:
    count_up(n-1)
    print(n)
```

count_up(2)

n 2

```
if n <= 0:
    pass
else:
    count_up(n-1)
    print(n)
```

Base case reached!

count_up(1)

n 1

```
if n <= 0:
    pass
else:
    count_up(n-1)
    print(n)
```

```
>>> count_up(4)
1
2
3
4
```

Recursion GOTCHAs!

GOTCHA #1

- If the problem that you are solving recursively **is not getting smaller**, that is, you are not getting closer to the base case --- **infinite recursion**!
- Never reaches the base case

```
def count_down_gotcha(n):
    '''Prints ints from 1 up to n'''
    if n == 1:  # Base case
        print(n)
    else:       # Recursive case
        print(n)
        count_down_gotcha(n)
```

Subproblem not getting smaller!

GOTCHA #2

- Missing base case/unreachable base case--- another way to cause **infinite recursion**!

```
def print_halves_gotcha(n):
    """Prints n, n/2, down to ... 1"""
    if n > 0:
        print(n)
        return print_halves_gotcha(n/2)
```

Always true!

"Maximum recursion depth exceeded"

- In practice, the infinite recursion examples will terminate when Python runs out of resources for creating function call frames, leads to a "maximum recursion depth exceeded" error message

Recursion vs. Iteration:

`sum_list`

sum_list

- **Goal:** Write a function to sum up a list of numbers
- Iterative approach? (i.e., using loops?)

Iterative Approach to `sum_list`

- **Goal:** Write a function to sum up a list of numbers
- Iterative approach:

```
def sum_list_iterative(num_lst):
    sum = 0
    for num in num_lst:
        sum += num
    return sum
```

```
>>> sum_list_iterative([3, 4, 20, 12, 2, 20])
61
```

sum_list

- **Goal:** Write a function to sum up a list of numbers
- Recursive approach?

Recursive approach to `sum_list`

- **Base case:**
  - `num_lst` is empty, return `0`
- **Recursive rule:**
  - Return first element of `num_lst` plus result from calling `sum_list` on rest of the elements of the list.
- Example: Suppose `num_lst = [6, 3, 6, 5]`
  - `sum_list([6, 3, 6, 5]) = 6 + sum_list([3, 6, 5])`
  - `sum_list([3, 6, 5]) = 3 + sum_list([6, 5])`
  - `sum_list([6, 5]) = 6 + sum_list([5])`
  - `sum_list([5]) = 5 + sum_list([])`
- For the base case we have `sum_list([])` returns `0`

Recursive approach to `sum_list`

- **Base case:**
  - `num_lst` is empty, return `0`
- **Recursive rule:**
  - Return first element of `num_lst` plus result from calling `sum_list` on rest of the elements of the list.

- Example: Suppose `num_lst = [6, 3, 6, 5]`
  - `sum_l` **20** `[6, 3, 6, 5]) = 6 +` `sum_l` **14** `[3, 6, 5])`
  - `sum_l` **14** `[3, 6, 5]) = 3 +` `sum_` **11** `t([6, 5])`
  - `sum_l` **11** `[6, 5]) = 6 + su` **5** `st([5])`
  - `sum_l` **5** `[5]) = 5 + s` **0** `ist([])`
- For the base case we have `sum_list([])` returns `0`

Recursive approach to `sum_list`

```
def sum_list(num_lst):
    """Returns sum of given list"""
    if not num_lst:
        return 0
    else:
        return num_lst[0] + sum_list(num_lst[1:])
```

```
>>> sum_list([3, 4, 20, 12, 2, 20])
61
```

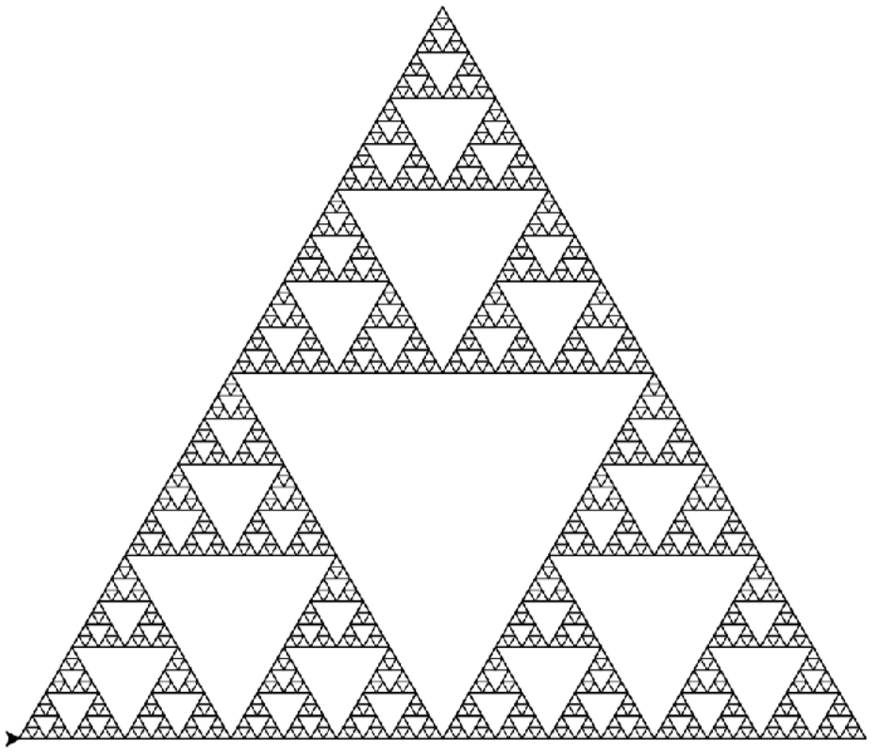
Compare `sum_list` approaches

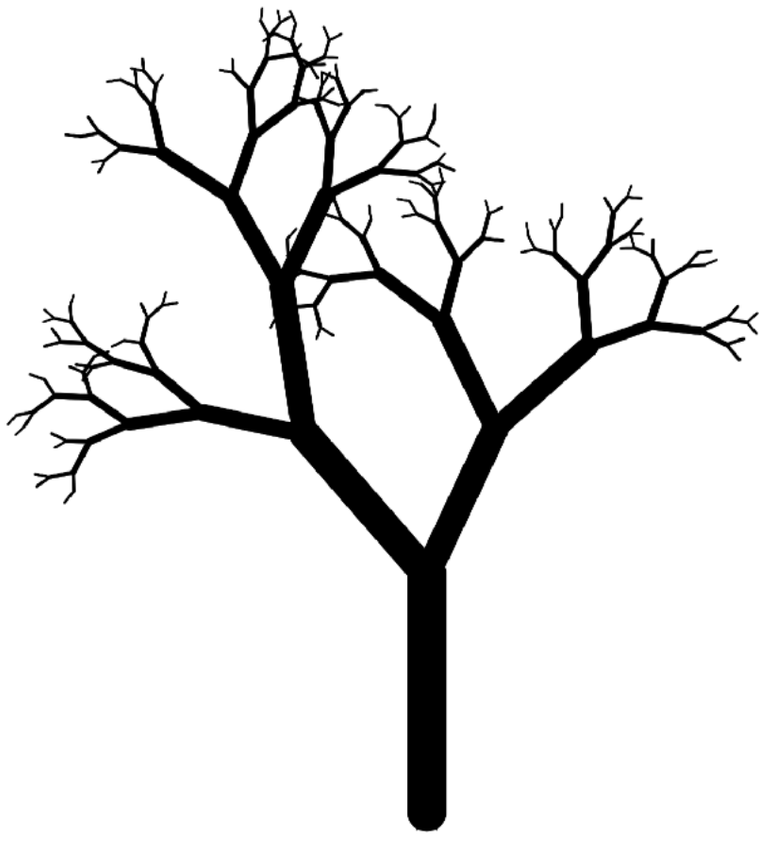
- Compare/Contrast:

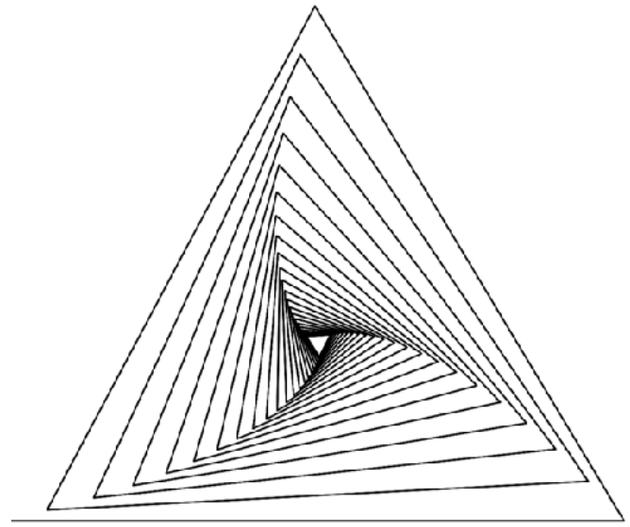
```
def sum_list_iterative(num_lst):
    sum = 0
    for num in num_lst:
        sum += num
    return sum
```

```
def sum_list(num_lst):
    if num_lst == []:
        return 0
    else:
        return num_lst[0] + sumList(num_lst[1:])
```

Graphical Recursion







The Turtle Module

- Turtle is a **graphics module** first introduced in the 1960s by computer scientists Seymour Papert, Wally Feurzig, and Cynthia Solomon.
- It uses a programmable cursor — fondly referred to as the "turtle" — to draw on a Cartesian plane (x and y axis.)

```
pen down
```

Turtle In Python

- **`turtle`** is available as a built-in module in Python. See the Python turtle module API for details.
- Basic turtle commands:

Use `from turtle import *` to use these commands

`fd(dist)`	turtle moves forward by dist
`bk(dist)`	turtle moves backward by dist
`lt(angle)`	turtle turns left angle degrees
`rt(angle)`	turtle turns right angle degrees
`up()`	(pen up) turtle raises pen in belly
`down()`	(pen down) turtle lowers pen from belly
`shape(shp)`	sets the turtle's shape to shp
`speed(spd)`	sets the turtle's speed 1-10 (slow-fast). 0 skips animation.
`home()`	turtle returns to (0,0) (center of screen)
`clear()`	delete turtle drawings; no change to turtle's state
`reset()`	delete turtle drawings; reset turtle's state
`setup(width, height)`	create a turtle window of given width and height

Basic Turtle Movement

- forward(dist) or fd(dist),
  left(angle) or lt(angle),
  right(angle) or rt(angle),
  backward(dist) or bk(dist)

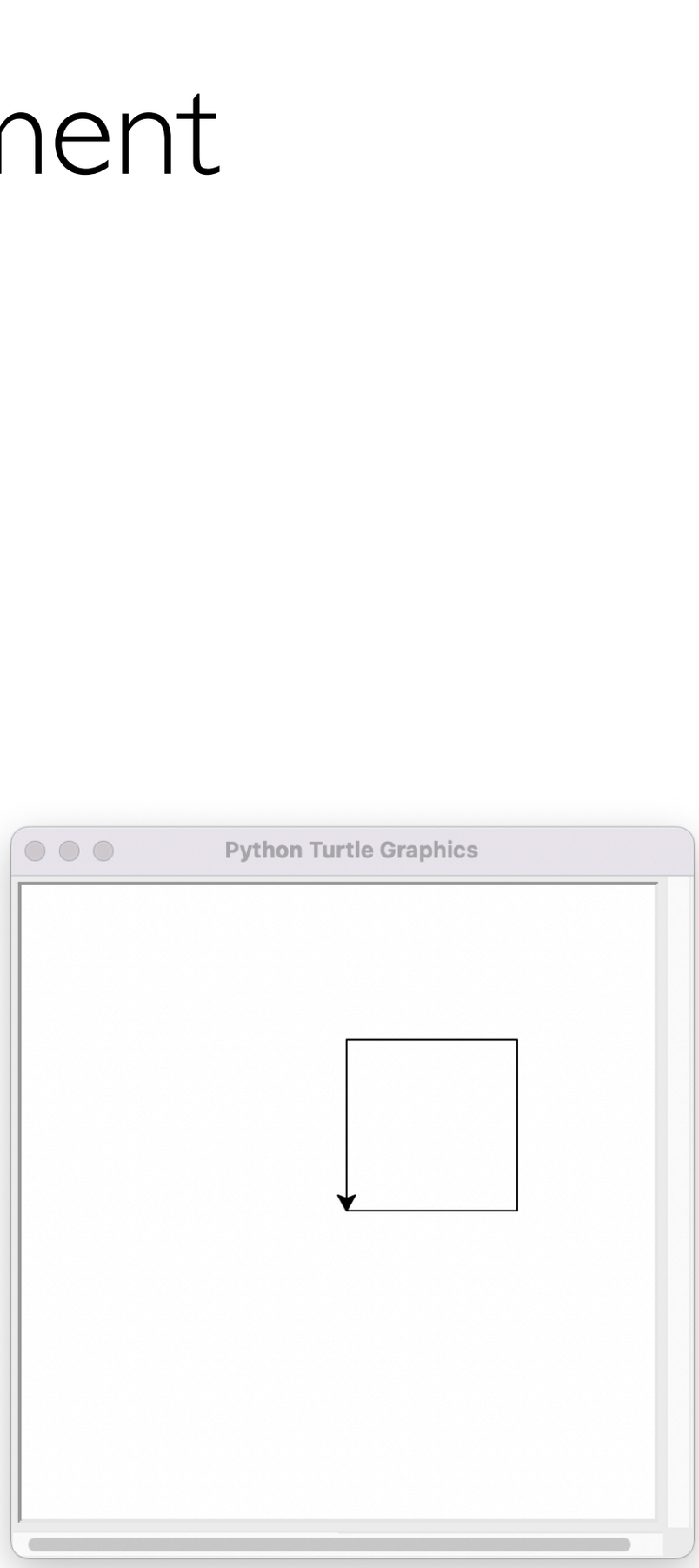
```
# set up a 400x400 turtle window
setup(400, 400)
reset()

fd(100) # move the turtle forward 100 pixels

lt(90) # turn the turtle 90 degrees to the left

fd(100) # move forward another 100 pixels

# complete a square
lt(90)
fd(100)
lt(90)
fd(100)
done()
```

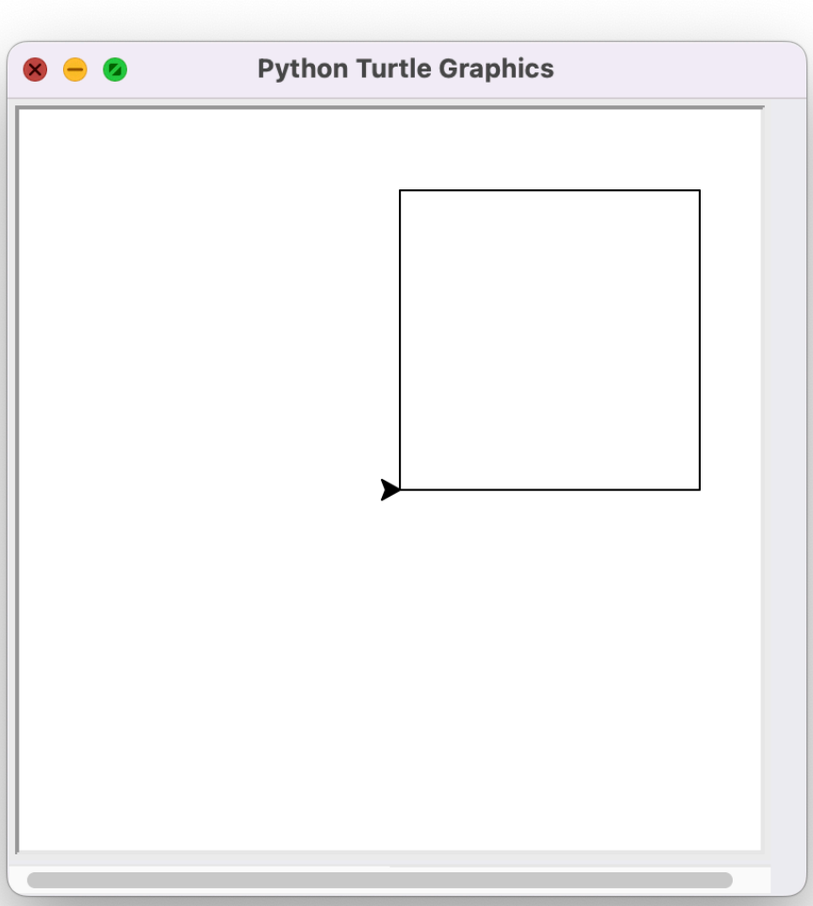


Drawing Basic Shapes With Turtle

- We can write functions that use turtle commands to draw shapes.
- For example, here's a function that draws a square of the desired size

```
def draw_square(length):
    # a loop that runs 4 times
    # and draws each side of the square
    for i in range(4):
        fd(length)
        lt(90)
    done()
```

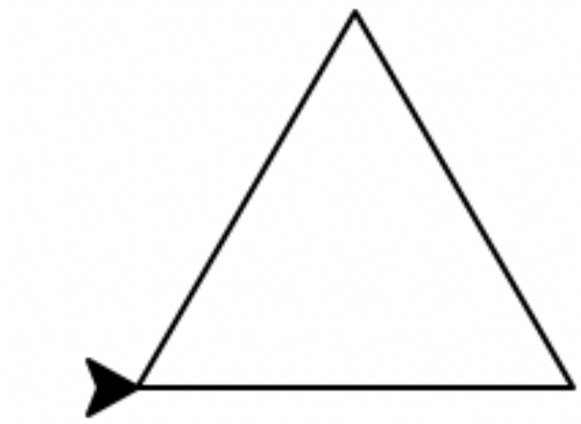
```
setup(400, 400)
reset()
draw_square(150)
```



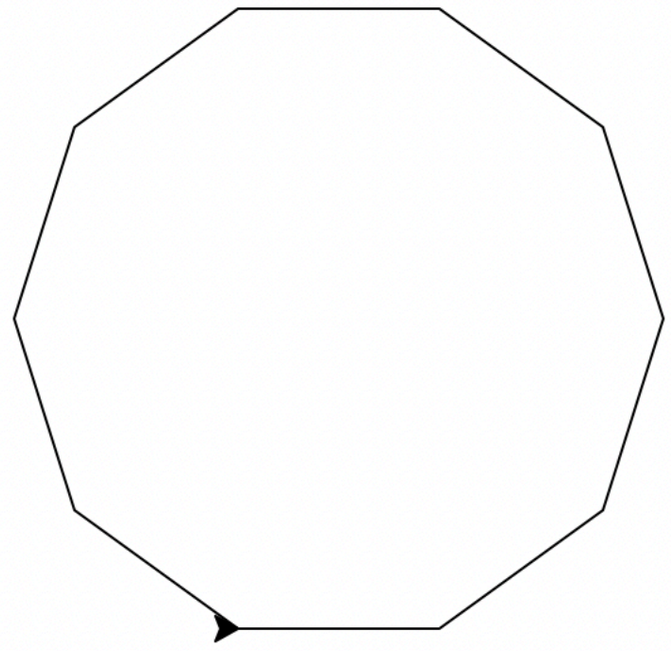
Drawing Basic Shapes With Turtle

- How about drawing polygons?

```
def draw_polygon(length, num_sides):
    for i in range(num_sides):
        fd(length)
        lt(360/num_sides)
    done()
```



```
draw_polygon(80, 3)
```



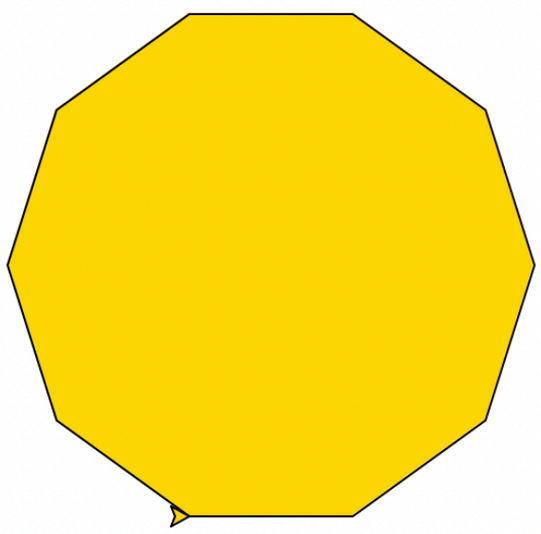
```
draw_polygon(80, 10)
```

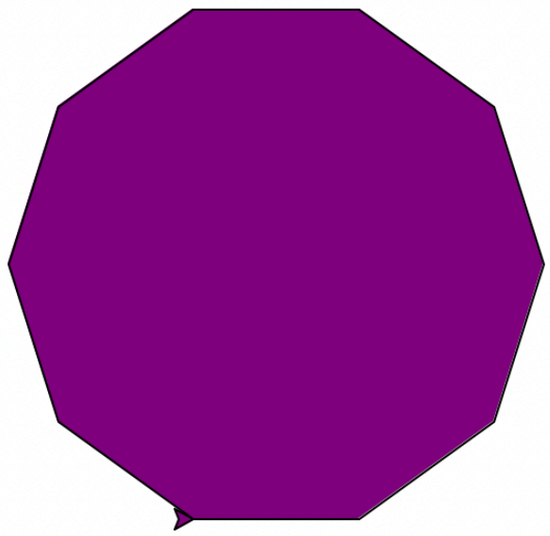
Adding Color!

- What if we wanted to add some color to our shapes?

```
def draw_polygon_color(length, num_sides, color):
    # set the color we want to fill the shape with
    # color is a string
    fillcolor(color)

    begin_fill()
    for i in range(num_sides):
        fd(length)
        lt(360/num_sides)
    end_fill()
    done()
```





```
draw_polygon_color(80, 10, "gold")
```

```
draw_polygon_color(80, 10, "purple")
```

Next Time: Recursive Figures With Turtle

- Next time we will explore how to draw recursive pictures with Turtle