CS134 Lecture 31:
Measuring Efficiency

Announcements & Logistics

- **HW 10** will be released today, due Mon @ 10 pm
  - Last HW
- **Lab 9 Boggle (Parts 1& 2)** due Wed/Thurs at 10 pm
  - Make sure your completed game satisfies all of the expected behavior mentioned in handout
  - Test your game thoroughly!
    - Not just "normal game behavior"
    - Stress test it with unexpected clicks, etc
- CS134 Scheduled Final: **Friday, May 17, 9:30 AM**

Do You Have Any Questions?

Last Time: Linked Lists

- Learned about linked lists
- Did a mix of list special methods using recursion and loops
  - Many more methods are possible: see code on course schedule

Today and Next Week

- Start discussing efficiency trade-offs surrounding certain operations, such as append and prepend, to a data type such as Linked List
- Introduce how we measure efficiency in Computer Science
- Discuss efficiency of some classic algorithms
    - Linear search
    - Binary search

Linked List Efficiency

- How can we compare the efficiency of the following LinkedList operations?
  - append an item at the end of a LinkedList
  - prepend an item to the beginning of a LinkedList
- Any thoughts on which is "faster" (without defining efficiency formally)
  - append needs to traverse the entire list to find last item
    - "number of steps" proportional to number of items
  - prepend just needs to change self._rest of newly inserted item
    - this is independent of how many items are in the LinkedList
- This is intuitively why append is more efficient than prepend
- For more formal discussion: need to figure out <u>what</u> we want to measure

Measuring Efficiency

Measuring Efficiency

- How do we measure the efficiency of our program?
  - We want programs that run "fast"
  - How should we measure this?
- One idea: use a stopwatch to see how long it takes
  - Reasonable proxy
  - But, what is it really measuring?
- Suppose I run the same program on a really slow/old computer vs a really powerful supercomputer
  - Stopwatch will measure different times!
  - Are we measuring how fast our program is or how fast the computer executes it?

Measuring Efficiency

- How do we measure the efficiency of our program?
  - We want programs that run "fast"
  - How should we measure this?
- One idea: use a stopwatch to see how long it takes
  - Measures how long a piece of code takes **on this machine on this particular input**
  - Machine (and input) dependent
- We want to isolate our ***program's efficiency***
  - How well does it scale to larger inputs?
  - How does it compare to other solutions to the same problem: which one is better?

Efficiency Metric: Goals

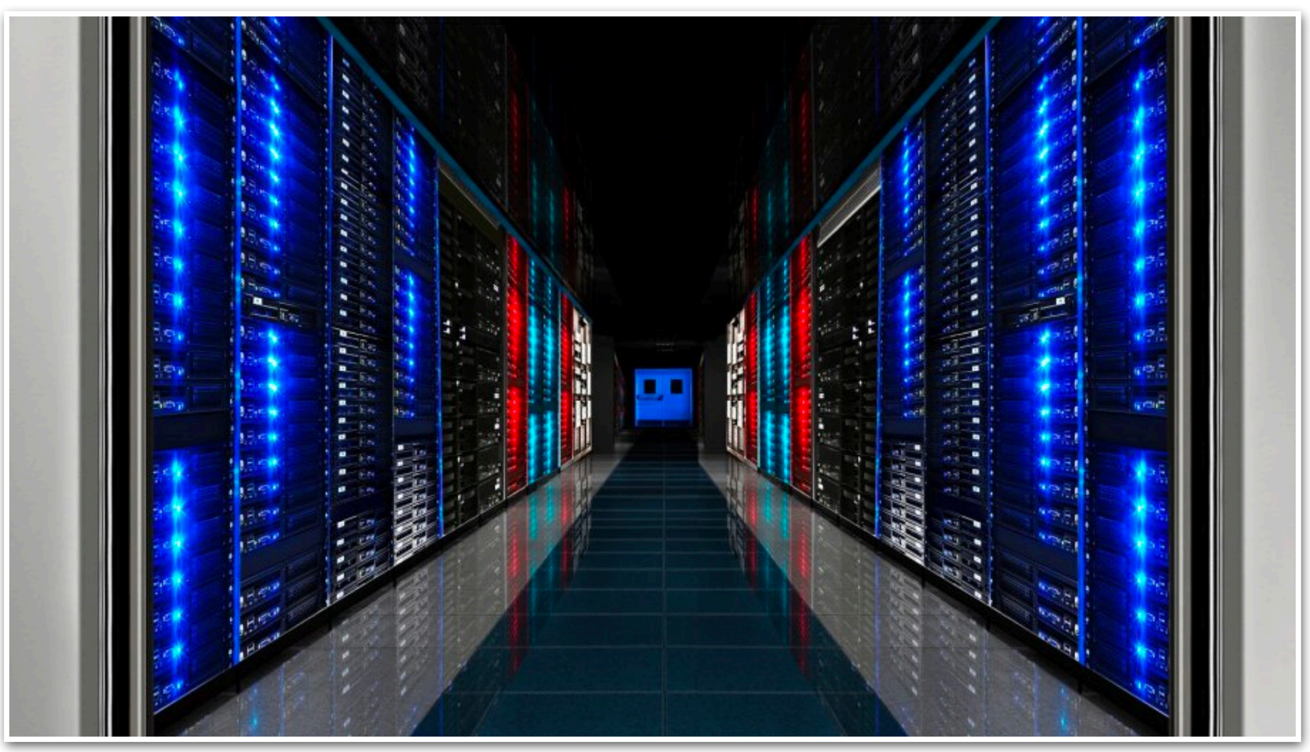
We want a method to evaluate efficiency that:

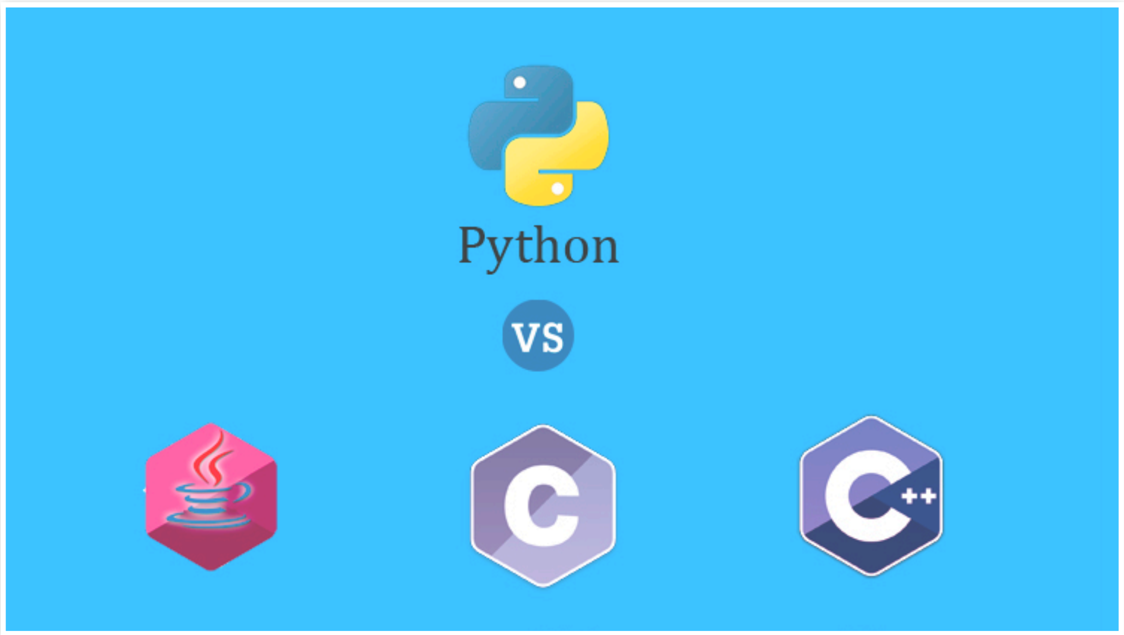
- **Is machine and language independent**
  - Analyze the ***algorithm*** (problem-solving approach)
- **Provides guarantees that hold for different types of inputs**
  - Some inputs may be "easy" to work with while others are not
- **Captures the dependence on input *size***
  - Determine how the performance "scales" when the input gets bigger
- **Captures the right level of specificity**
  - We don't want to be too specific (cumbersome)
  - Measure things that matter, ignore what doesn't

Platform/Language Independent

Machine and language independence

- We want to evaluate how good the algorithm is, rather than how good the machine or implementation is
- Basic idea: Count the **number of steps** taken by the algorithm
- Sometimes referred to as the "running time"



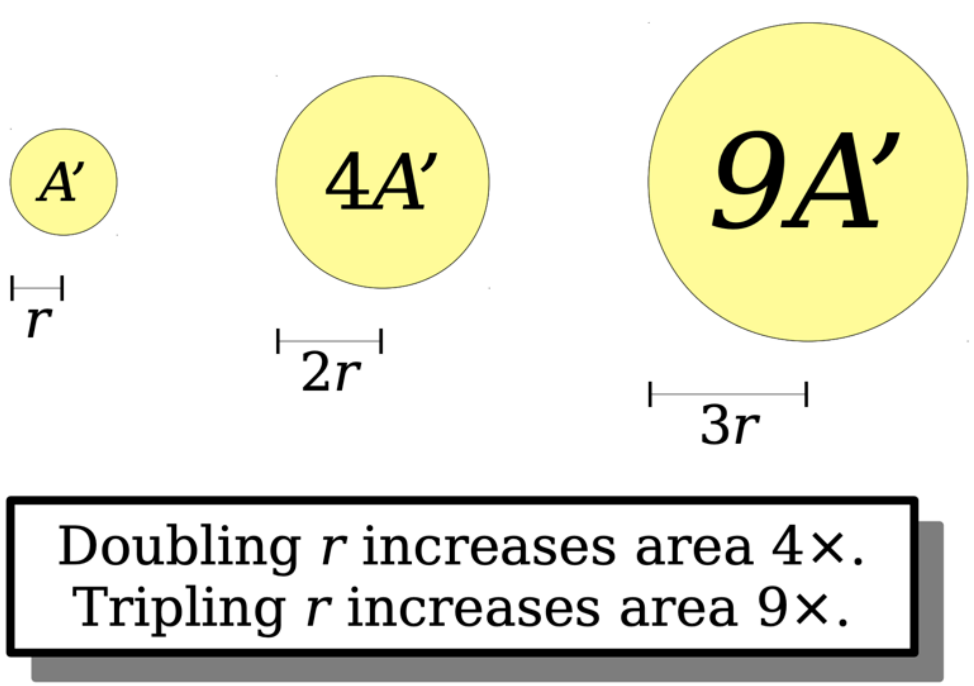


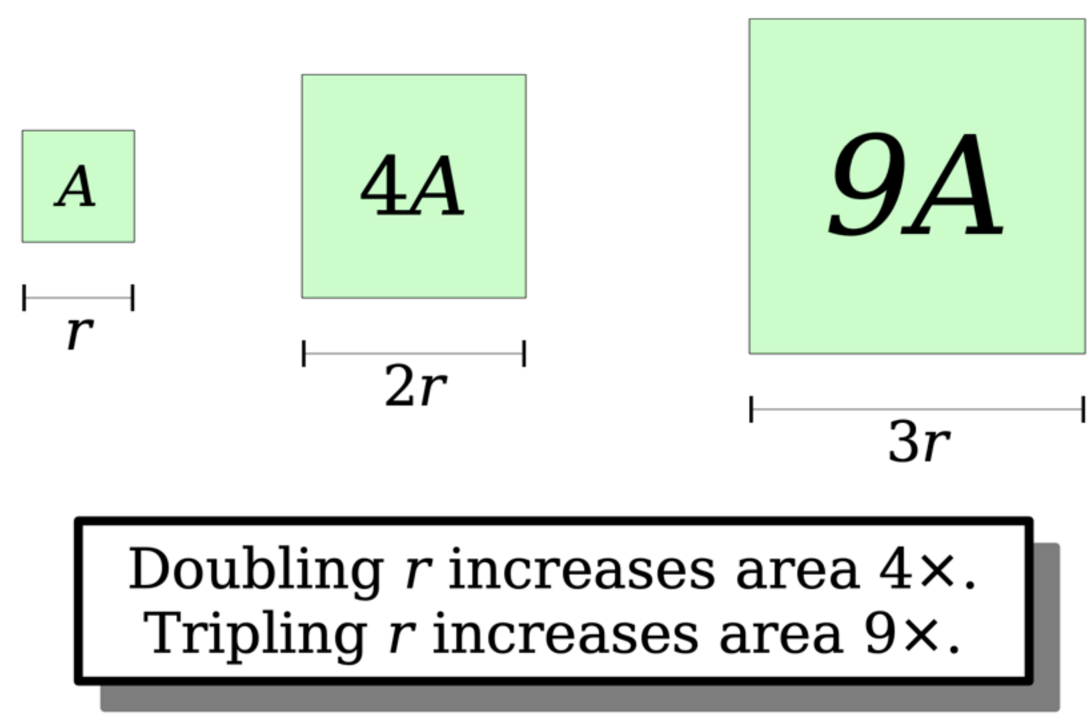
Worst-Case Analysis

- We can't just analyze our algorithm on a few inputs and declare victory. Which of the following is the scenario we should be measuring?
  - **Best case.** Minimum number of steps taken over all possible inputs of a given size
  - **Average case.** Average number of steps taken over all possible inputs of a given size
  - **Worst case.** Maximum number of steps taken over all possible inputs of a given size.
- Benefits of worst case analysis:
  - Regardless of input size, we can conclude that the algorithm always does *at least as well as* the pessimistic analysis

Dependence on Input Size

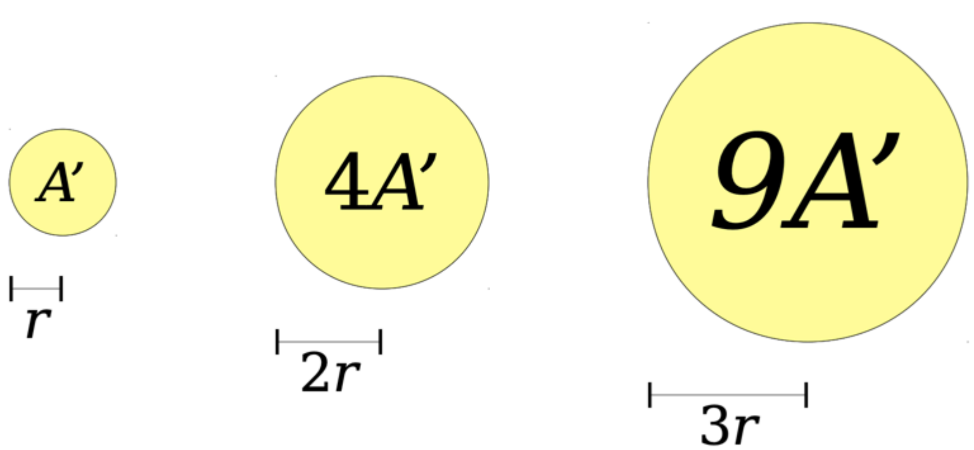
- We generally don't care about performance on "small inputs"
- Instead we care about "the rate at which the completion time grows" with respect to the input size
- For example, consider the area of a square or circle: while the formula for each is different, they both grow at the same rate wrt radius
    - doubling radius increases area by 4x, tripling increases by 9x



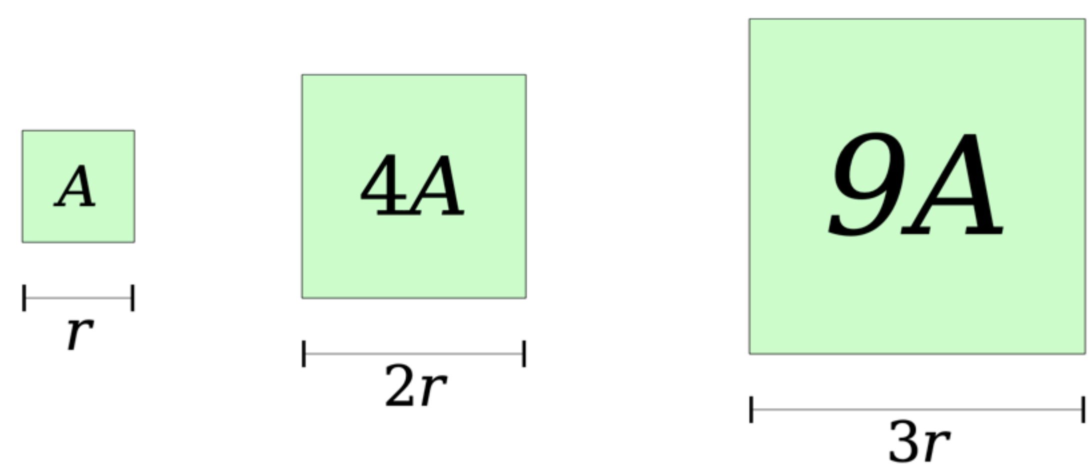


Dependence on Input Size: Big-O

- Big-O notation in Computer Science is a way of quantifying (in fact, upper bounding) the growth rate of algorithms/functions wrt input size
- For example:
    - A square of side length r has area $O(r^2)$.
    - A circle of radius r has area $O(r^2)$.



Doubling r increases area $4\times$.
Tripling r increases area $9\times$.



Doubling r increases area $4\times$.
Tripling r increases area $9\times$.

Dependence on Input Size: Big-O

- Big-O notation captures the **rate** at which the **number of steps taken** by the algorithm **grows** wrt size of input n, "as n gets large"
- Not precise by design, it ignores information about:
  - Constants (that do not depend on input size n), e.g. $100n = O(n)$
  - Lower-order terms: terms that contribute to the growth but are not dominant: $O(n^2 + n + 10) = O(n^2)$
- Powerful tool for predicting performance behavior: focuses on what matters, ignores the rest
- Separates fundamental improvements from smaller optimizations
- Won't study this notion too formally: covered in CS136 and CS256!

Append vs Prepend: Big Oh

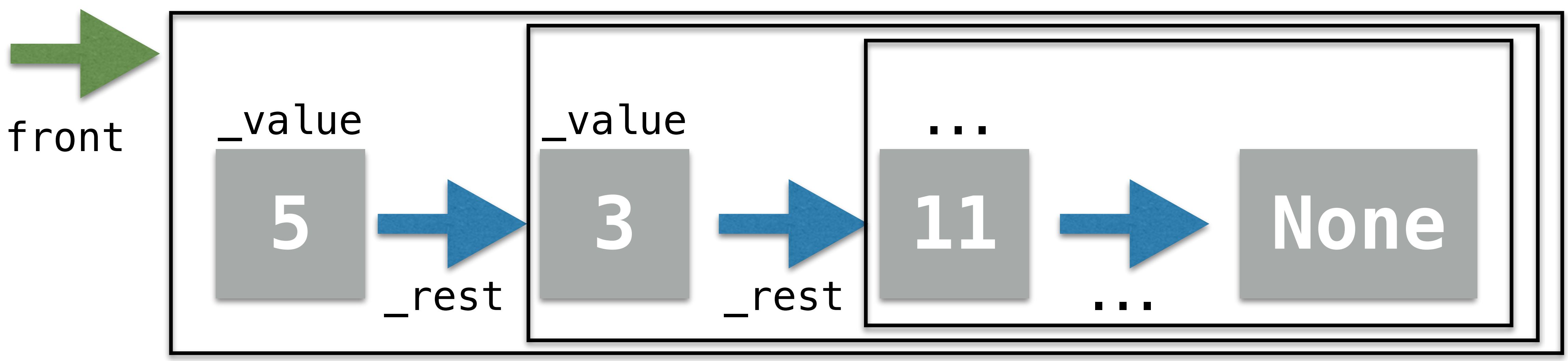
- Let's revisit append vs prepend efficiency
- How does the cost of append grow with number of items in LinkedList?
  - Need to traverse `len(LinkedList)` items at least
  - Grows linearly with input size
- How does the cost of prepend grow with number of items in LinkedList?
  - Independent of input size!
  - We call this $O(1)$ or constant time:
    - Essentially saying does not grow as input size gets large

Lists (Arrays) vs. Linked Lists

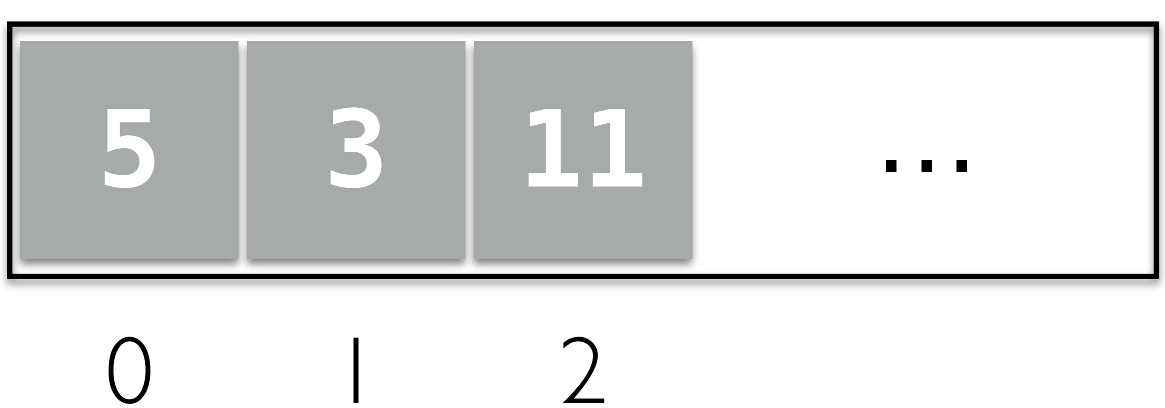
Efficiency Trade Offs

Lists vs Linked Lists

- **Linked Lists**: "pointer-based" data structure, items need not be contiguous in memory

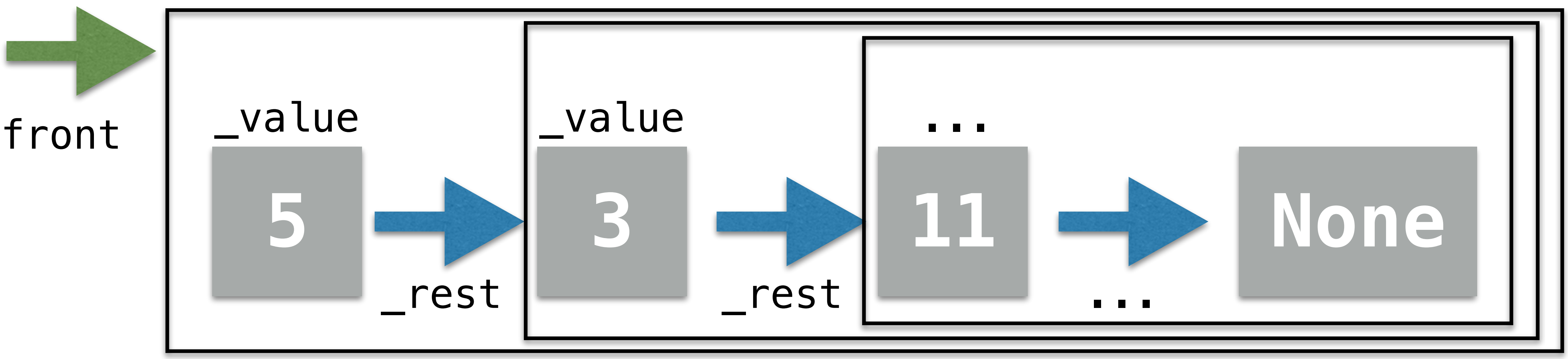


- **Arrays:** index-based data structure items are always stored contiguously in memory (think of a Python built-in list as an array)



Lists vs Linked Lists

- **Linked Lists**: Can grow and shrink on the fly: do not need to know size at the time of creation (therefore no wasted space!)



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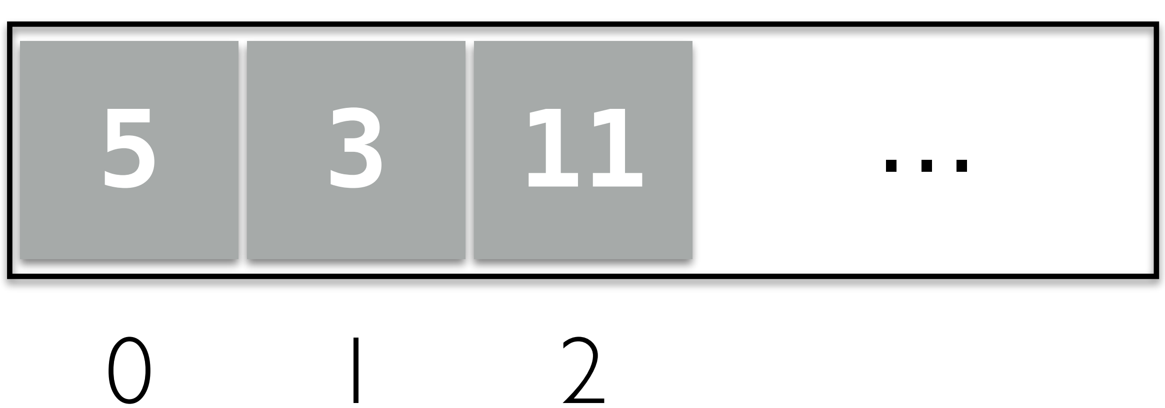
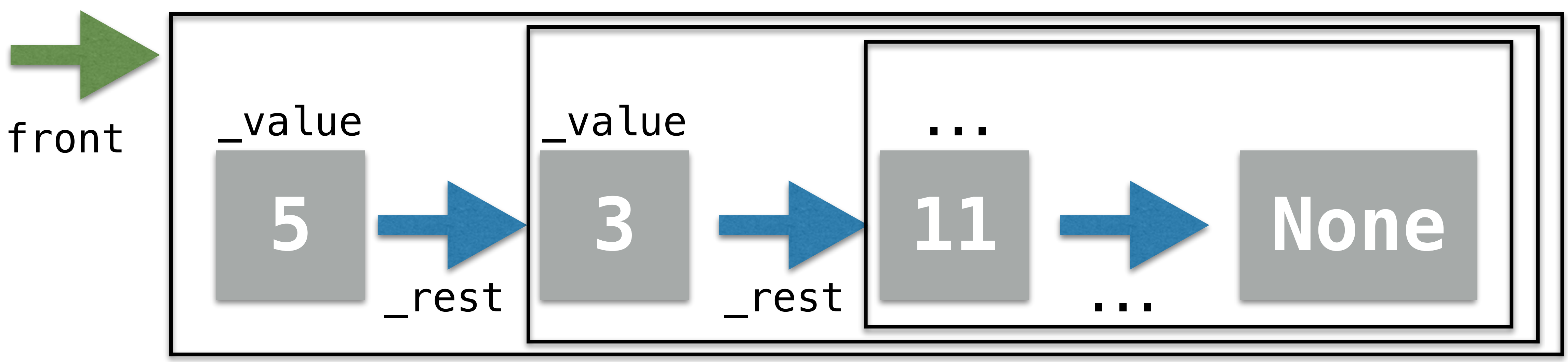
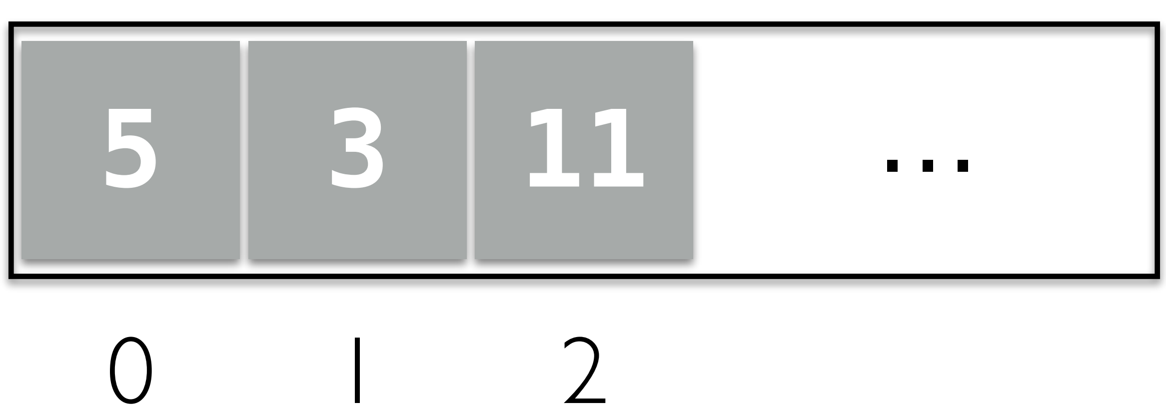


Array vs Linked Lists

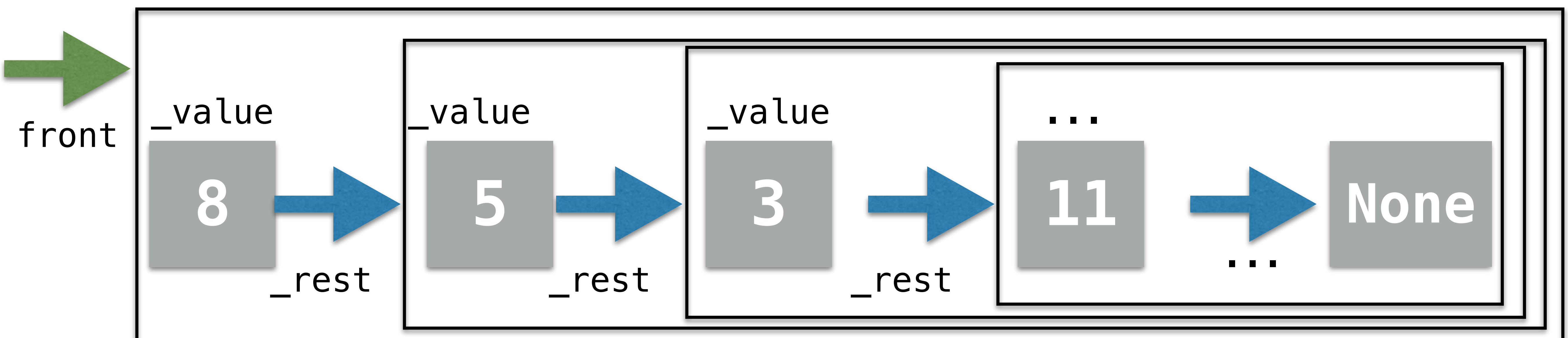
- Inserts at the beginning: which one is better?





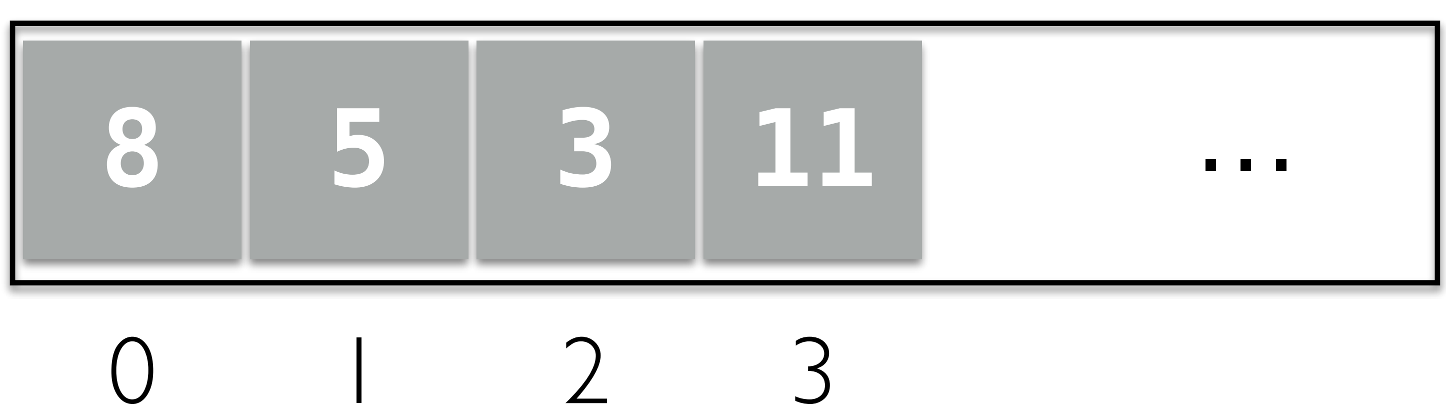
Array vs Linked Lists

- Linked list steps:
    - Point front to new element
    - Point rest of new element to old list
    - These steps don't depend on size of list
    - Therefore, run-time is **constant**, that is, $O(1)$ time



Array vs Linked Lists

- Now consider an array-based list
- To insert at index 0, we need to shift every element over by one spot
  - This takes time proportional to the size: linear time or $O(n)$
- So when are arrays more efficient?
  - When **indexing** elements: they give **direct access** $O(1)$
  - Linked list: we need to traverse the list to get to the element $O(n)$



So Which is Better?

- It depends!
- Think about what operations are a priority in your program!
    - We should choose the way to represent/organize our data according to how we plan to use the data
- Let's take an example of an application where one of the data structures is way more efficient than the other

Searching in a Sequence

Search

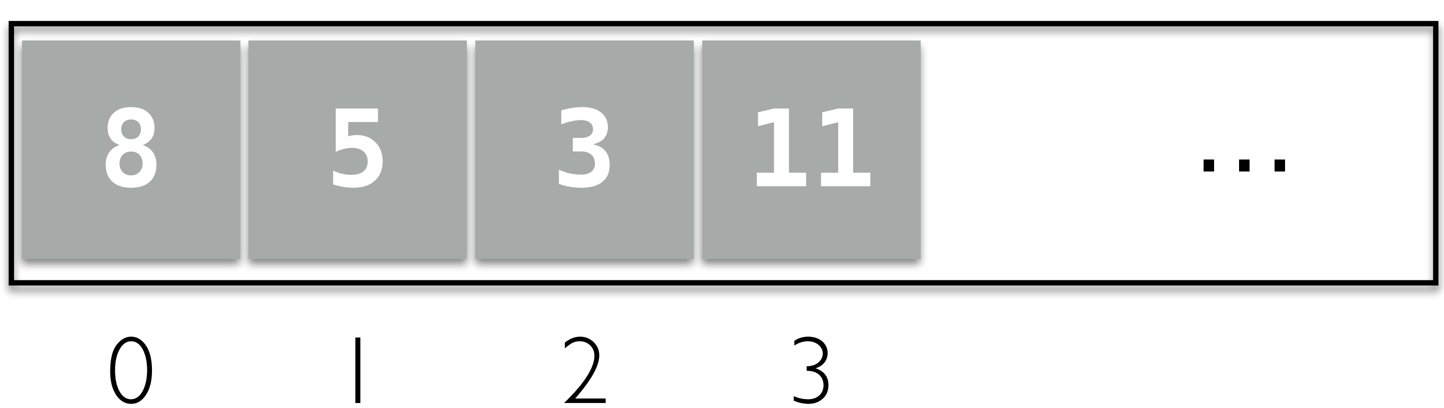
- **Search.** Given an input sequence `seq`, search if a given `item` is in the sequence.
    - For example, if a name is in a sequence of student names
- **Input:** a sequence of n items and a query item
    - For now suppose this can be in ***any order***
- **Output:** True if query item is in sequence, else False
- Can use `in` operator to do this (calls `__contains__`)
    - But without knowing how it works, can't analyze efficiency
- Let's figure out a direct way to solve this problem

Searching in a Sequence

- First algorithm: iterate through the items in sequence and compare each item to query

```
def linear_search(item, seq):
    for elem in seq:
        if elem == item:
            return True
    return False
```

Might return early if item is first elem in seq, but we are interested in the **worst case analysis**; this happens if item is not in seq at all



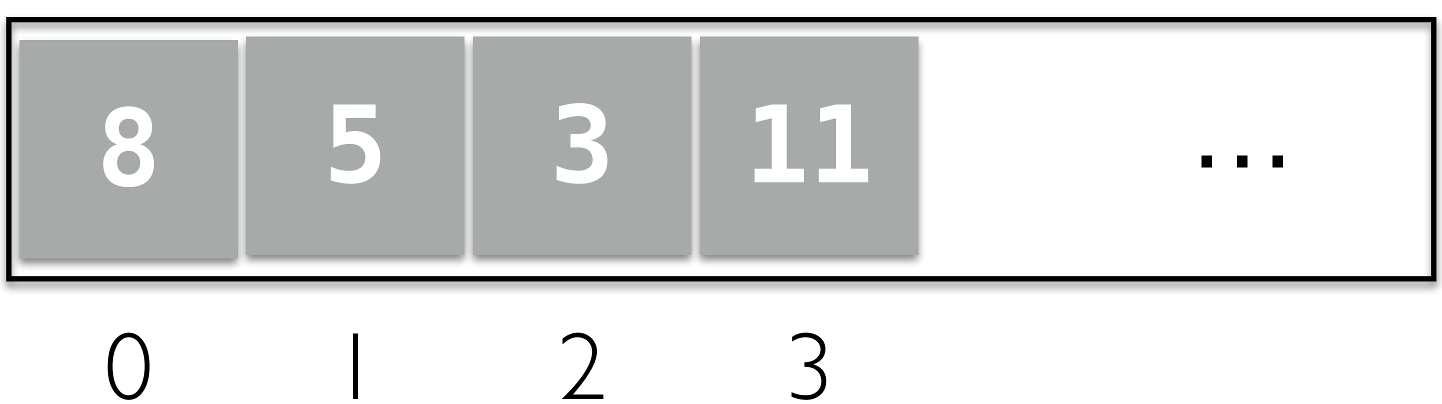
Searching in a Sequence

- In the worst case, we have to walk through the entire sequence
- Overall, the number of steps is linear in n : we write $O(n)$ in Big Oh

```python
def linear_search(item, seq):
    for elem in seq:
        if elem == item:
            return True
    return False
```

Loop runs n items in worst case

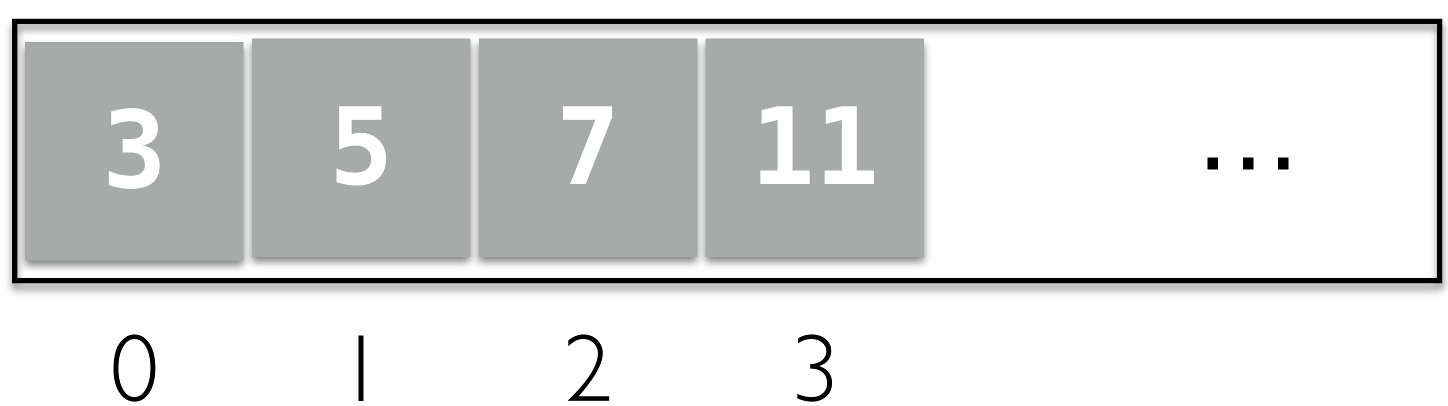
One equality check per iteration: assume comparing elem == item is one step



Searching in an Array

- Can we do better?
  - Not if the elements are in arbitrary order
- What if the sequence is **sorted**?
  - Can we utilize this somehow and search more efficiently?

How do we search for an item (say 10) in a **sorted** array?



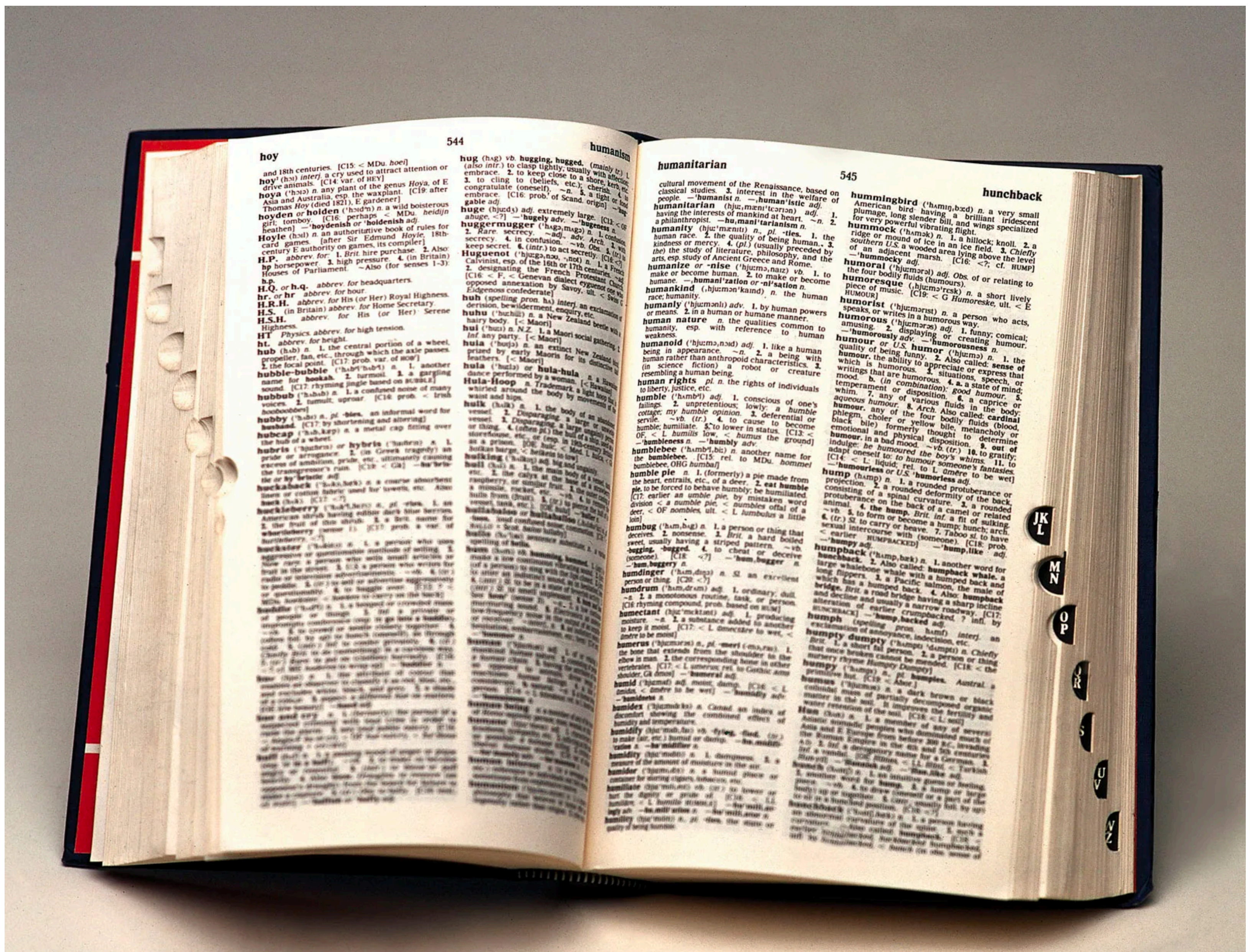
Let’s Play a Game

- I’m thinking of a number between 0 and 100…
  - If you guess a number, I’ll tell you either:
    - You’ve guessed my number!
    - My number is larger than your guess
    - My number is smaller than your guess
- What is your guessing strategy?

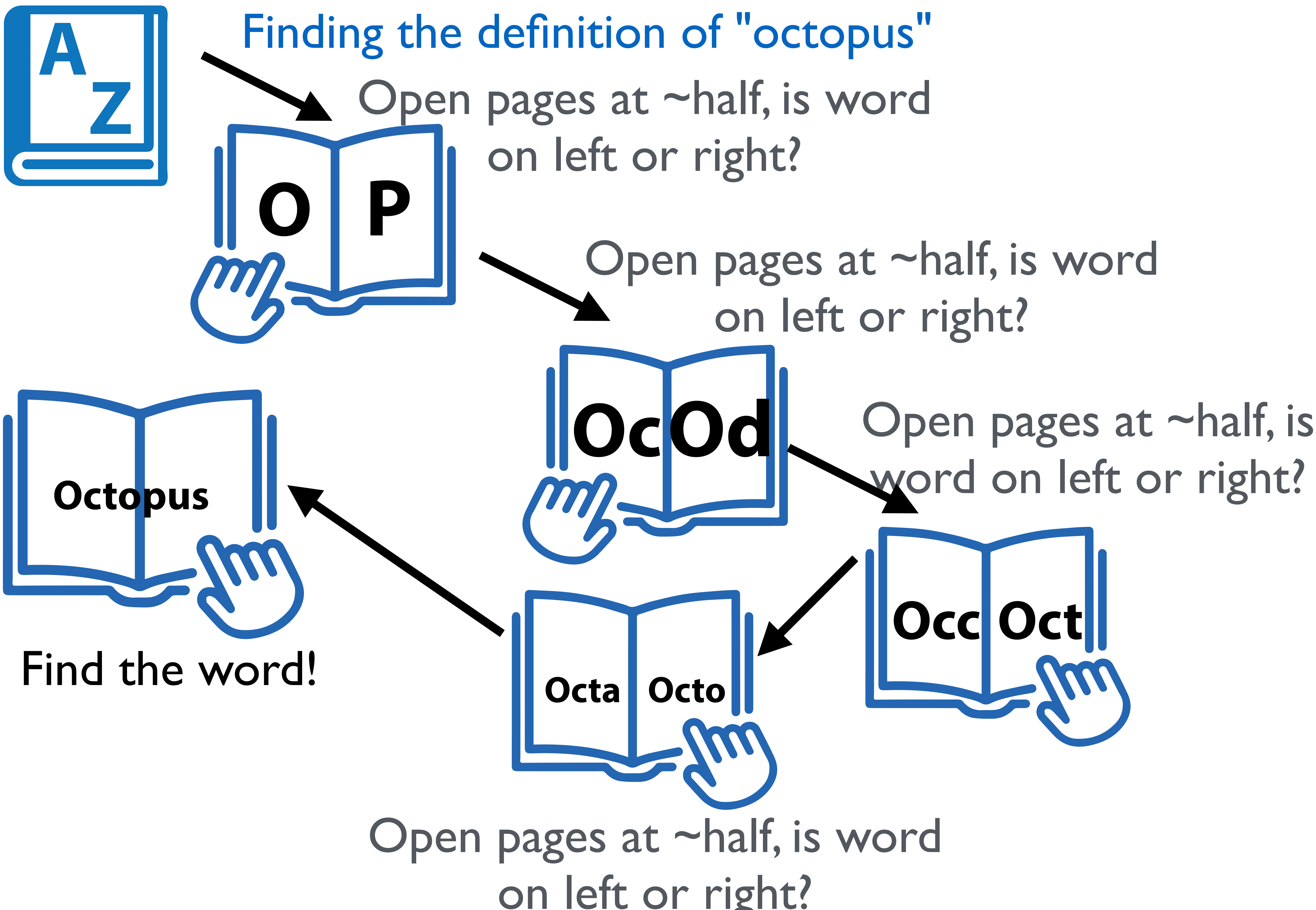
- What if I picked a number between 0 and 1 million?

Example: Dictionary

- How do we look up a word in a (physical) dictionary?
- Words are listed in alphabetical order



Example: Dictionary

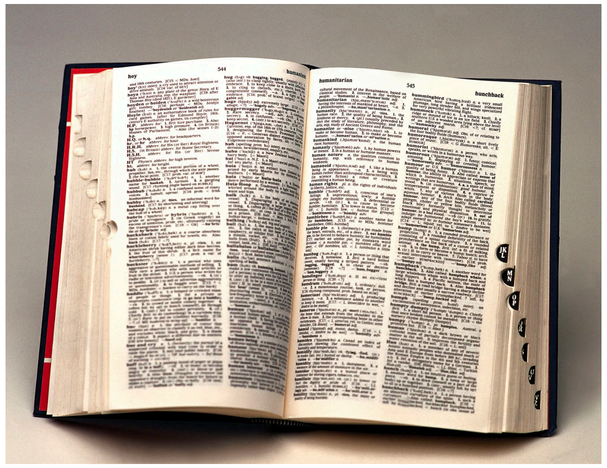


How Good is This Method?

- **Goal:** Analyze # pages we need to look at until we find the word
- We want the worst case: possible to get lucky and find the word right on the middle page, but we don't want to consider luck!
- Each time we look at the "middle" of the remaining pages, the number of pages we need to look at is divided by 2
- A 1024-page dictionary requires at most 11 lookups:
  1024 pages, < 512, <256, <128, <64, <32, <16, <8, <4, <2, <1 page.
- Only needed to look at 11 pages out of 1024 !
- **Challenge**: What if we have an n page dictionary, what function of n characterizes the (worst-case) number of lookups?

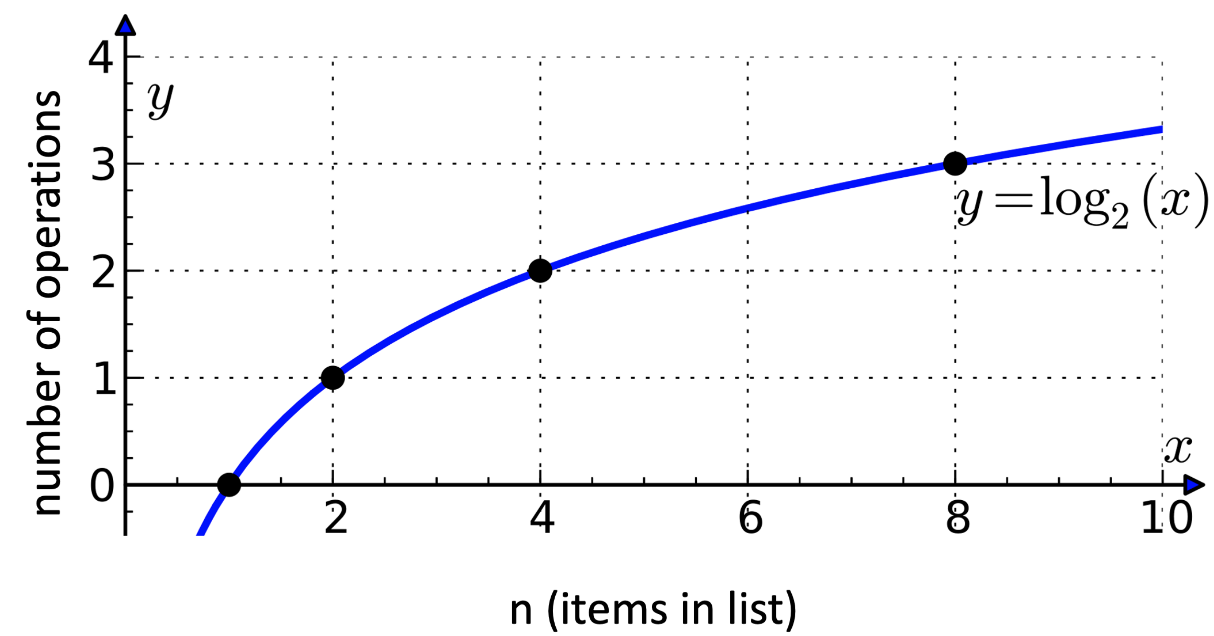
Logarithms: our favorite function

- Logarithms are the inverse function to exponentiation
- $\log_2 n$ describes the exponent to which 2 must be raised to produce n
- That is, $2^{\log_2 n} = n$
- Alternatively:
  - $\log_2 n$ (essentially) describes the number of times n must be divided by 2 to reduce it to 1 or below
- For us, here's the important takeaway:
  - How many times can we divide n by 2 until we get down to 1
  - $\approx \log_2 n$



O(log n)

- When you double the number of elements, it only increases the number of operations by 1
- 2 items in the list, 1 operation
  - log 2 = 1
- When you have 4 items, increases operations to 2
  - log 4 = 2
- When you have 8 items, only 3 operations
  - log 8 = 3



Binary Search

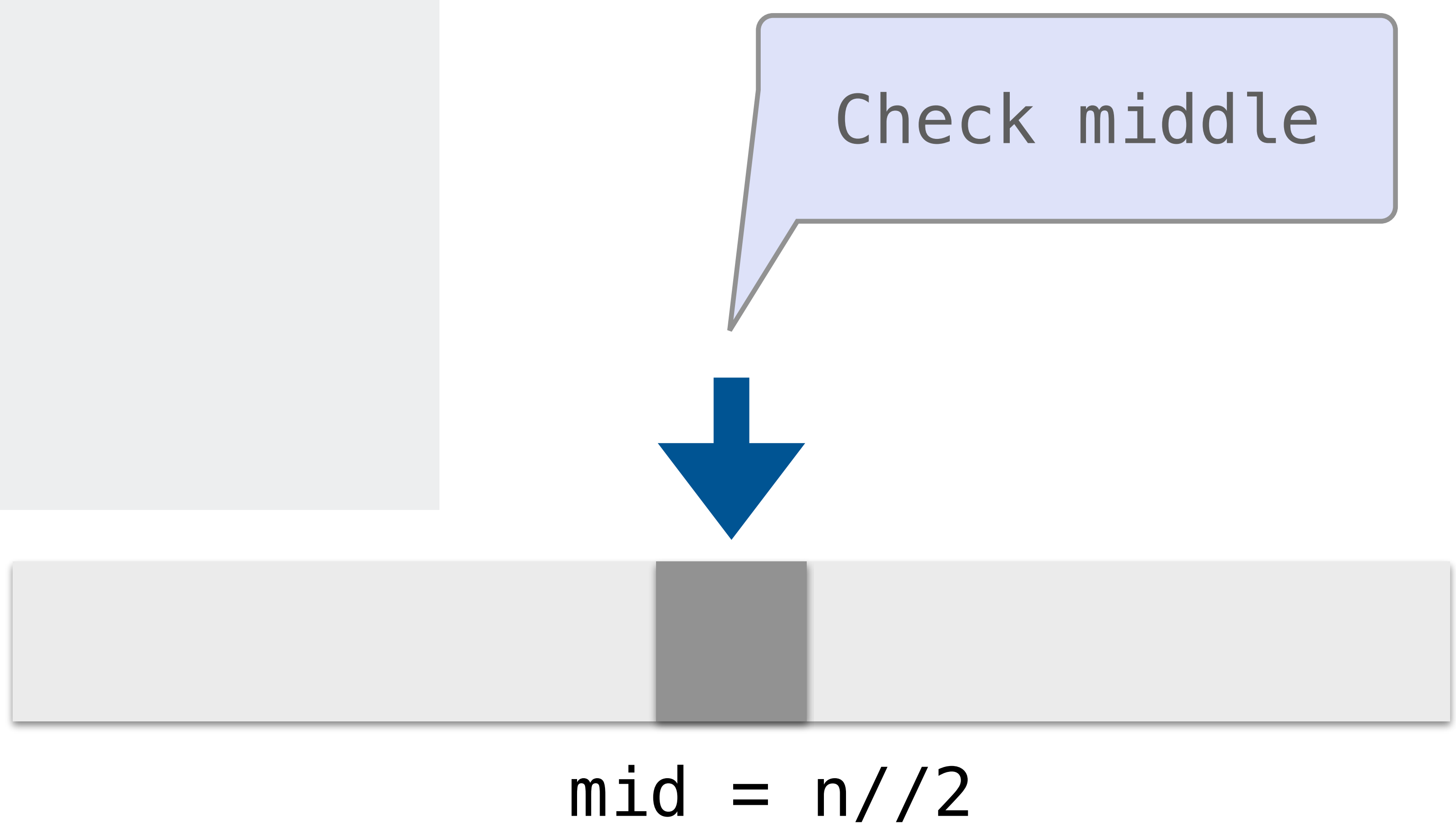
- The **recursive search algorithm** we described to search in a sorted array is called **binary search**
- It can be much more efficient than a **linear search**
  - Takes $\approx \log n$ lookups if we can index into sequence efficiently
- Which data structure supports fast access/indexing?
  - Accessing an item at index i in an array requires constant time
  - Accessing an item at index i in a LinkedList can require traversing the whole list (even if it is sorted!): linear time
- To get a more efficient search algorithm, it is not only important to use the right algorithm, we need to use the right data structure as well!

Binary Search

- Base cases? When are we done?
    - If list is too small (or empty) to continue searching
    - If item we're searching for is the middle element

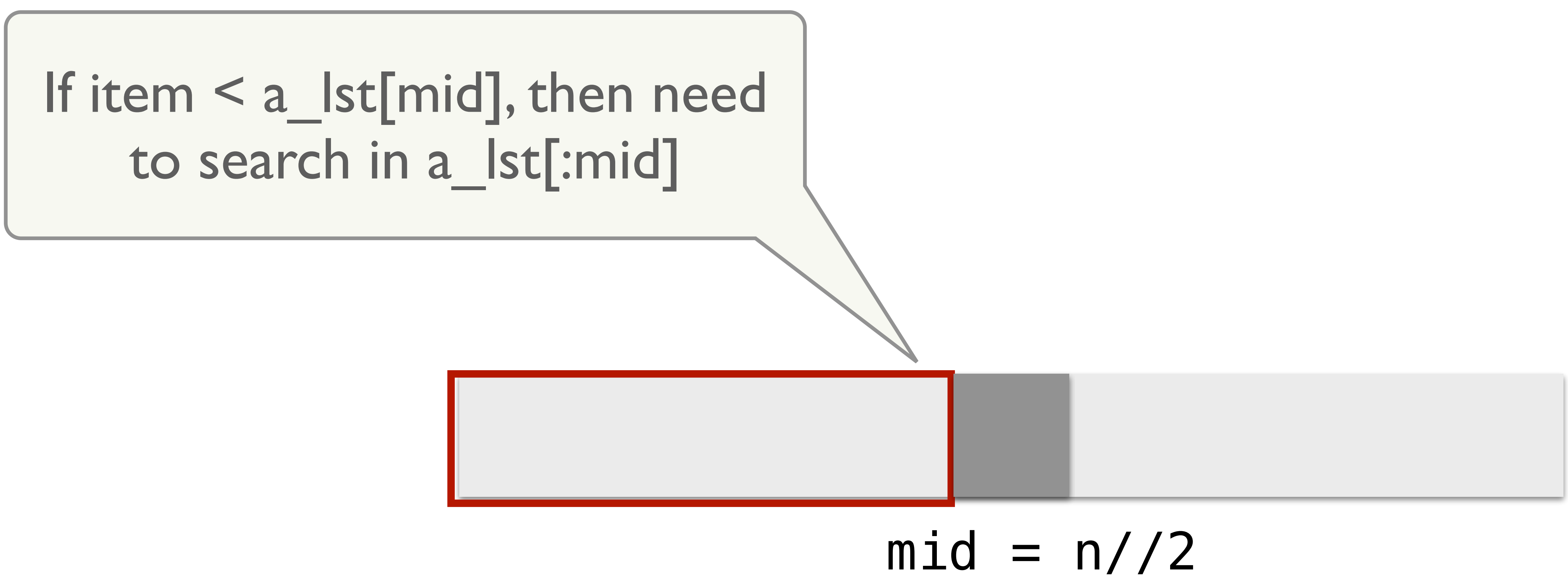
```python
def binary_search(a_lst, item):
    """Assume a_lst is sorted."""
    n = len(a_lst)
    mid = n // 2
    # base case 1
    if n == 0:
        return False
    # base case 2
    elif item == a_lst[mid]:
        return True

    # recursive cases...
```



Binary Search

- Recursive case:
    - Recurse on left side if item is smaller than middle
    - Recurse on right side if item is larger than middle



Binary Search

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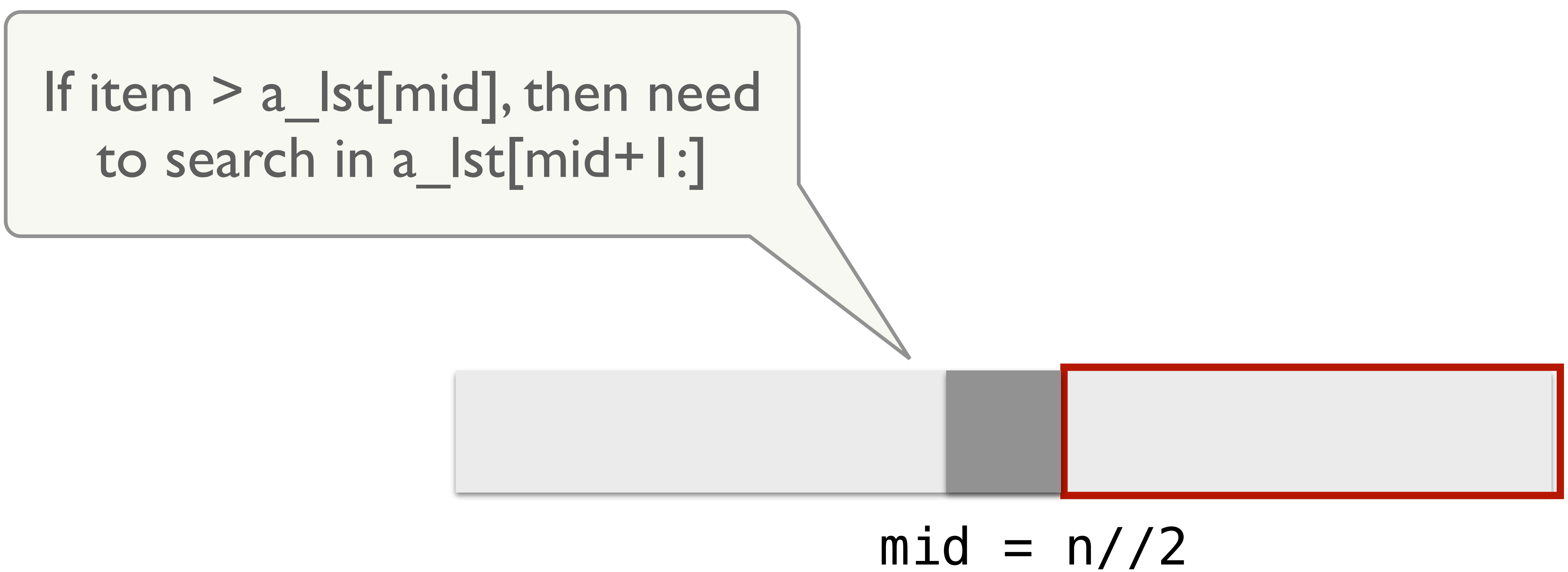


Binary Search

```python
def binary_search(a_lst, item):
    """ Assume a_lst is sorted."""
    n = len(a_lst)
    mid = n // 2
    # base case 1
    if n == 0:
        return False
    # base case 2
    elif item == a_lst[mid]:
        return True

    # recurse on left
    elif item < a_lst[mid]:
        return binary_search(a_lst[:mid], item)
    # recurse on right
    else:
        return binary_search(a_lst[mid + 1:], item)
```

Technically, there is one *small* problem with our implementation. List splicing is actually O(n)!

Binary Search: Better

```
def binary_search_better(a_lst, item, index_start, index_end):
    """ Assume a_lst is sorted."""

    n = index_end - index_start
    mid = (n // 2) + index_start
    # base case 1
    if n <= 0:
        return False
    # base case 2
    elif item == a_lst[mid]:
        return True

    # recurse on left
    elif item < a_lst[mid]:
        return binary_search_better(a_lst, item, 0, mid)
    # recurse on right
    else:
        return binary_search_better(a_lst, item, mid+1, index_end)
```

Passing start/end indices as arguments avoids the need to splice!

Big-O Notation

- Tells you how fast an algorithm is / the run-time of algorithms
  - But not in seconds!
- Tells you how fast the algorithm grows in number of operations

O(log n)

"Big O" Number of Operations

Understanding Big-O

- Notation: n often denotes the number of elements (size)
- **Constant time** or $O(1)$: when an operation does not depend on the number of elements, e.g.
  - Addition/subtraction/multiplication of two values, or defining a variable etc are all constant time
- **Linear time** or $O(n)$: when an operation requires time proportional to the number of elements, e.g.:

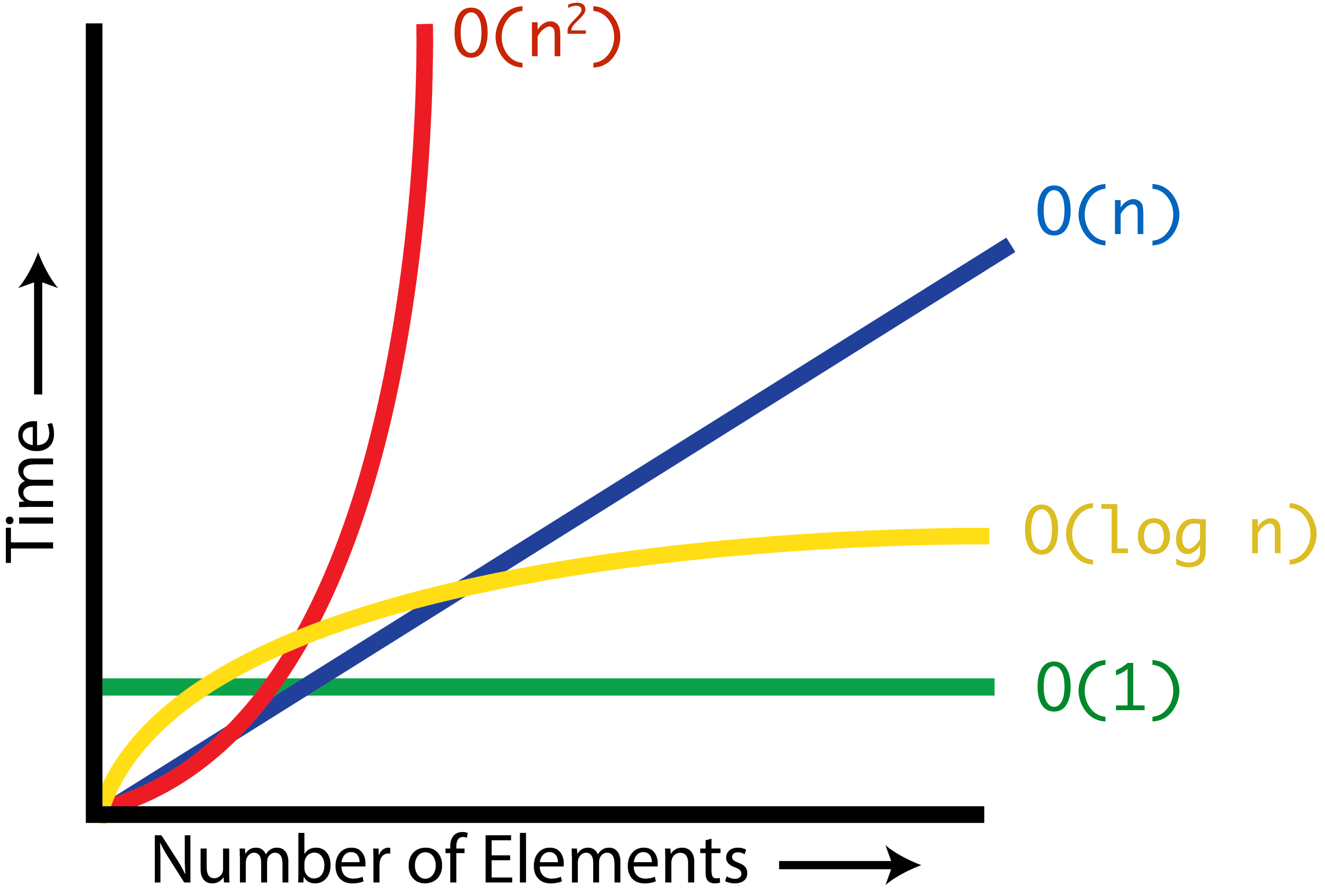
```
for item in seq:
    <do something>
```

- **Quadratic time** or $O(n^2)$: nested loops are often quadratic, e.g.,

```
for i in range(n):
    for j in range(n):
        <do something>
```

Big-O: Common Functions

- Notation: n often denotes the number of elements (size)
- Our goal: understand efficiency of some algorithms at a high level



Binary Search

- Applying binary search

```
>>> binary_search(['Artie', 'Chels', 'Linus', 'Pixel', 'Sally', 'Wally'], 'Pixel')
True
>>> binary_search(['Artie', 'Chels', 'Linus', 'Pixel', 'Sally', 'Wally'], 'Scooby')
False
```

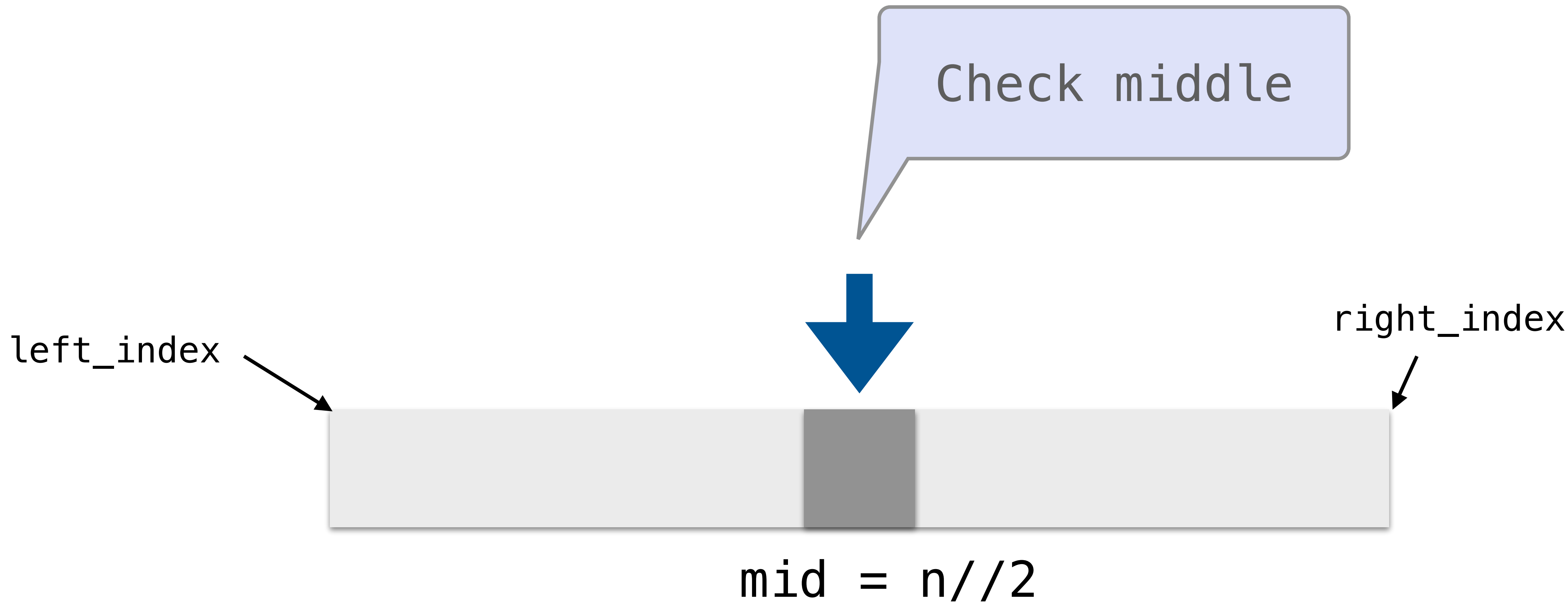
Binary Search with Flamenco Dancers

https://www.youtube.com/watch?v=wz7XgKowJlg

The end!

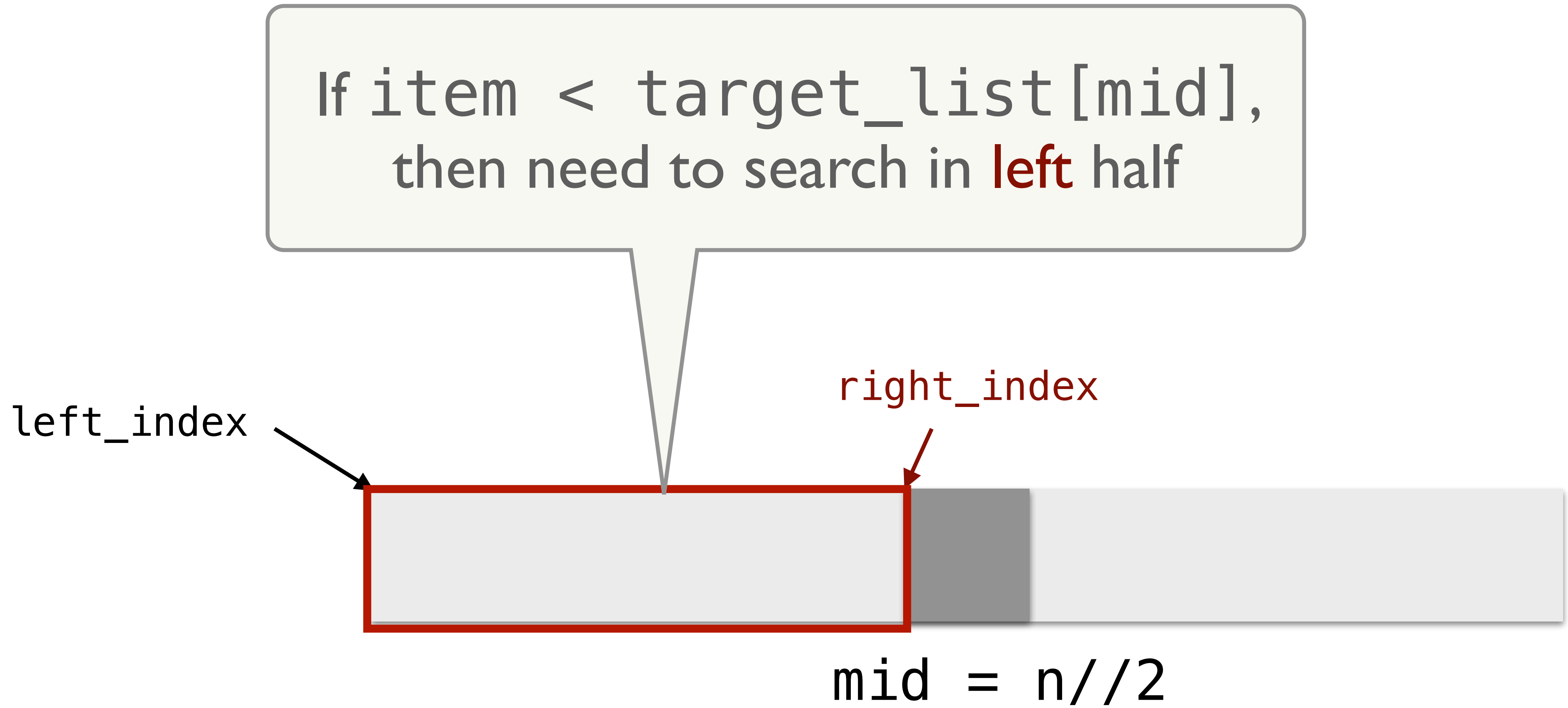
Leftover Slides

Binary Search



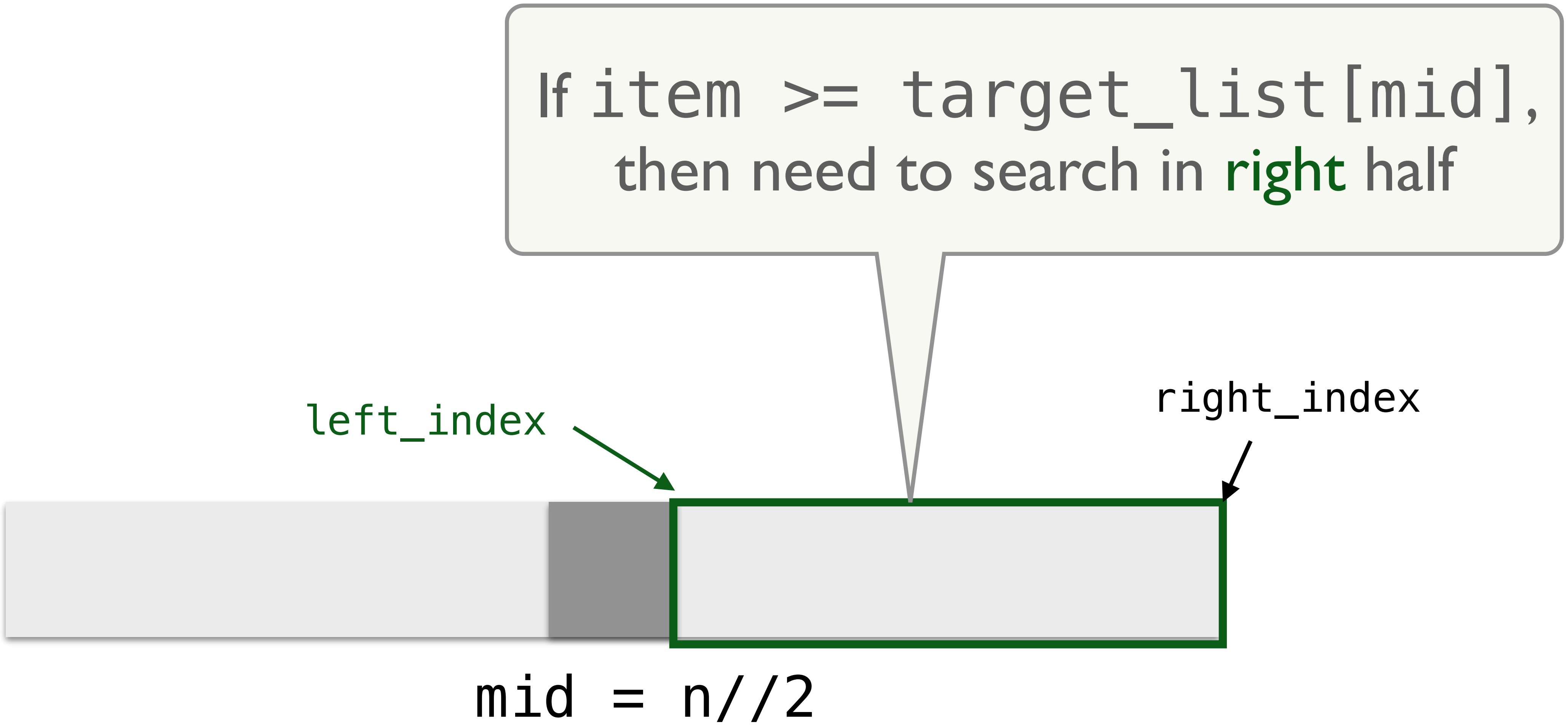
- If `target_list[mid] == item`, we're done and we return True

Binary Search



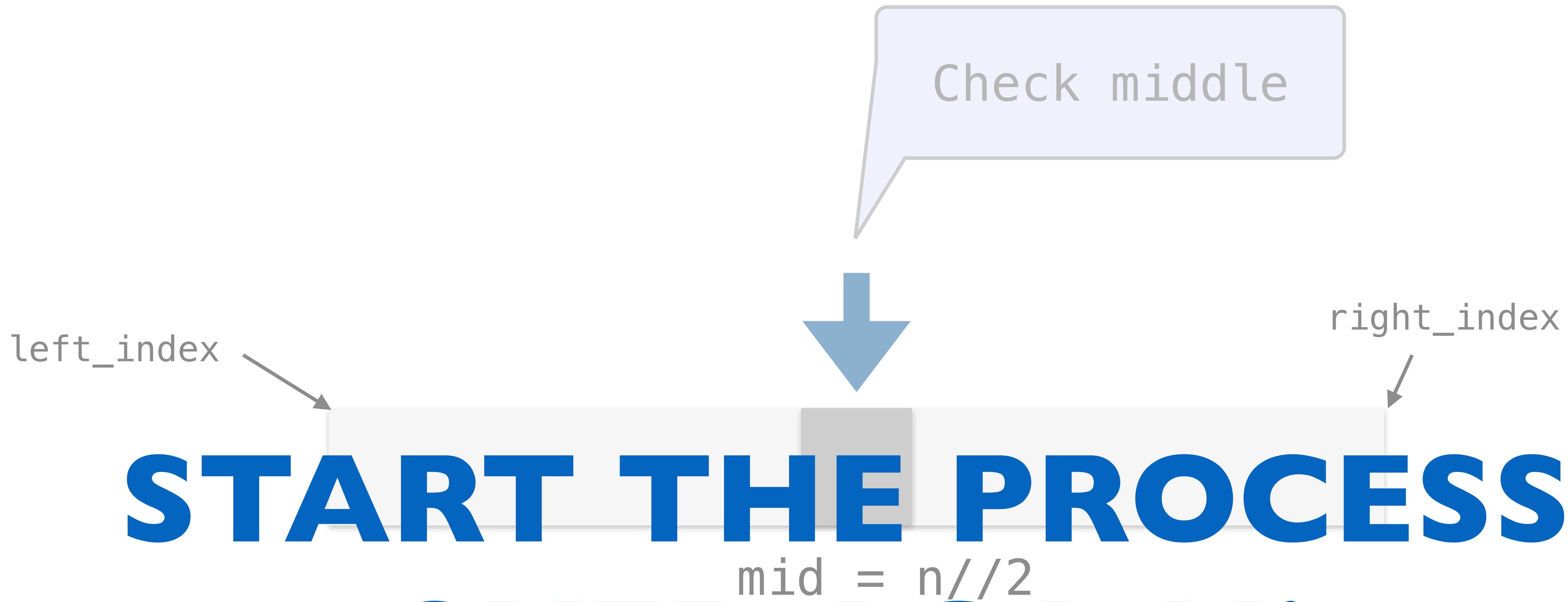
- Update the `right_index`!

Binary Search



- Update the `left_index`!

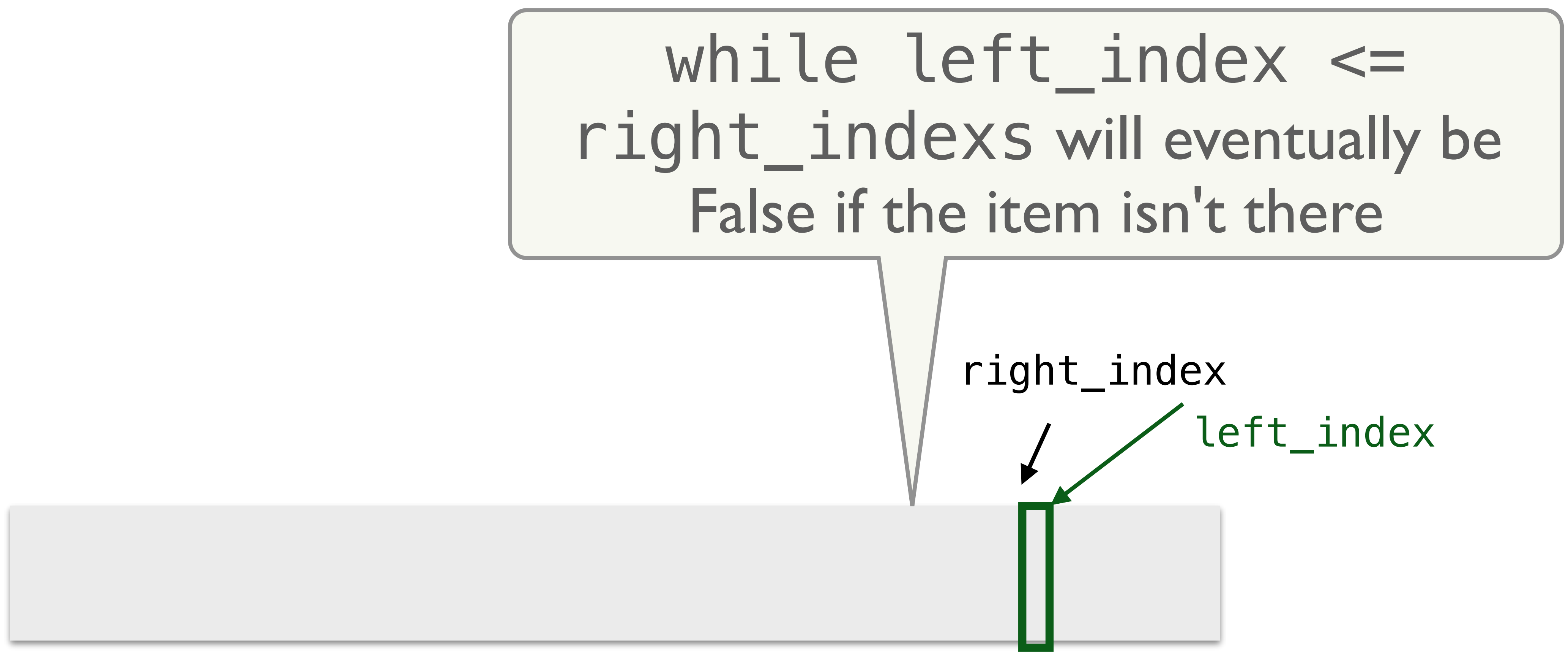
Binary Search



START THE PROCESS OVER AGAIN!

- If `target_list[mid] == item`, we're done and we return True

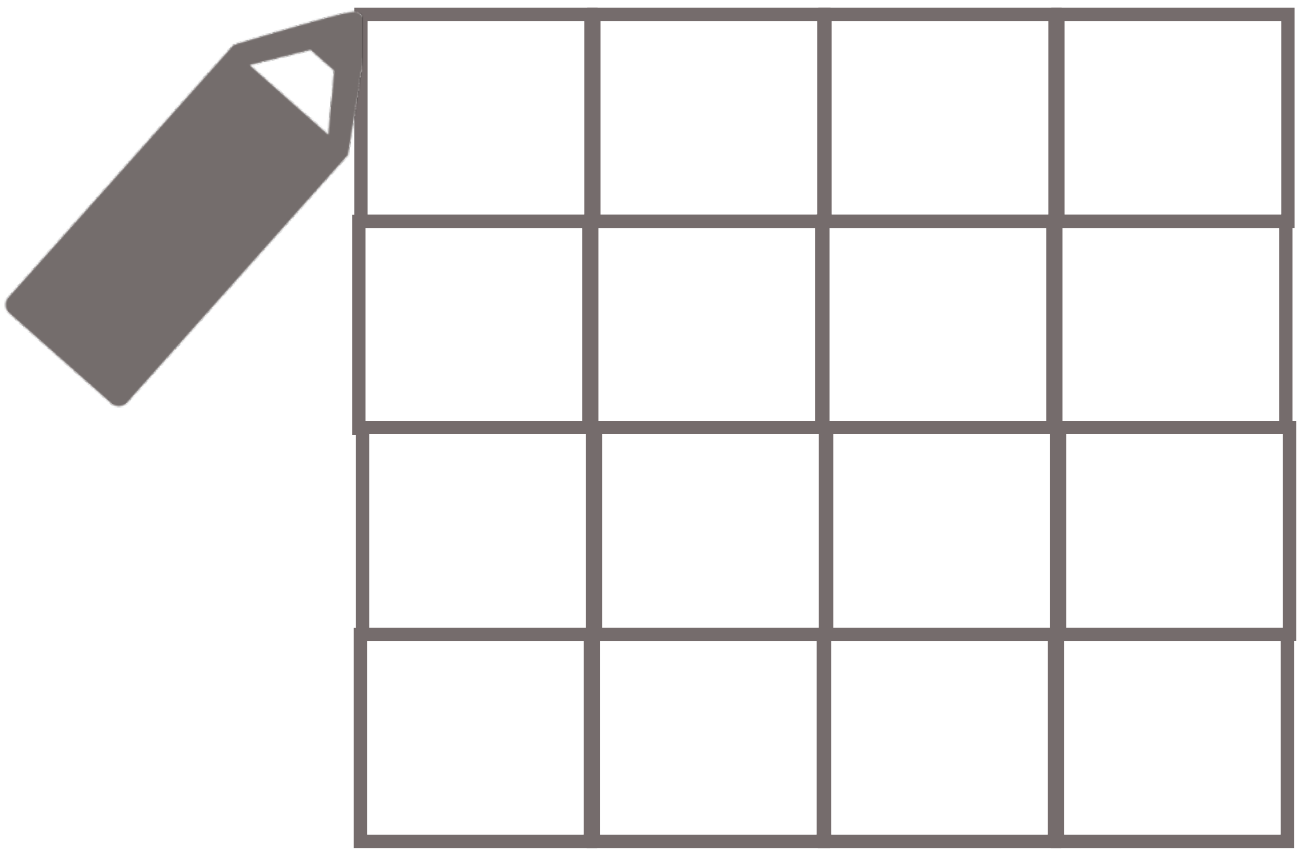
Binary Search



- `left_index` will be incremented repeatedly (due to the >=) and will eventually be greater than `right_index`

Logarithmic Intuition

Suppose we want to create a 4x4 grid on a piece of paper:



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

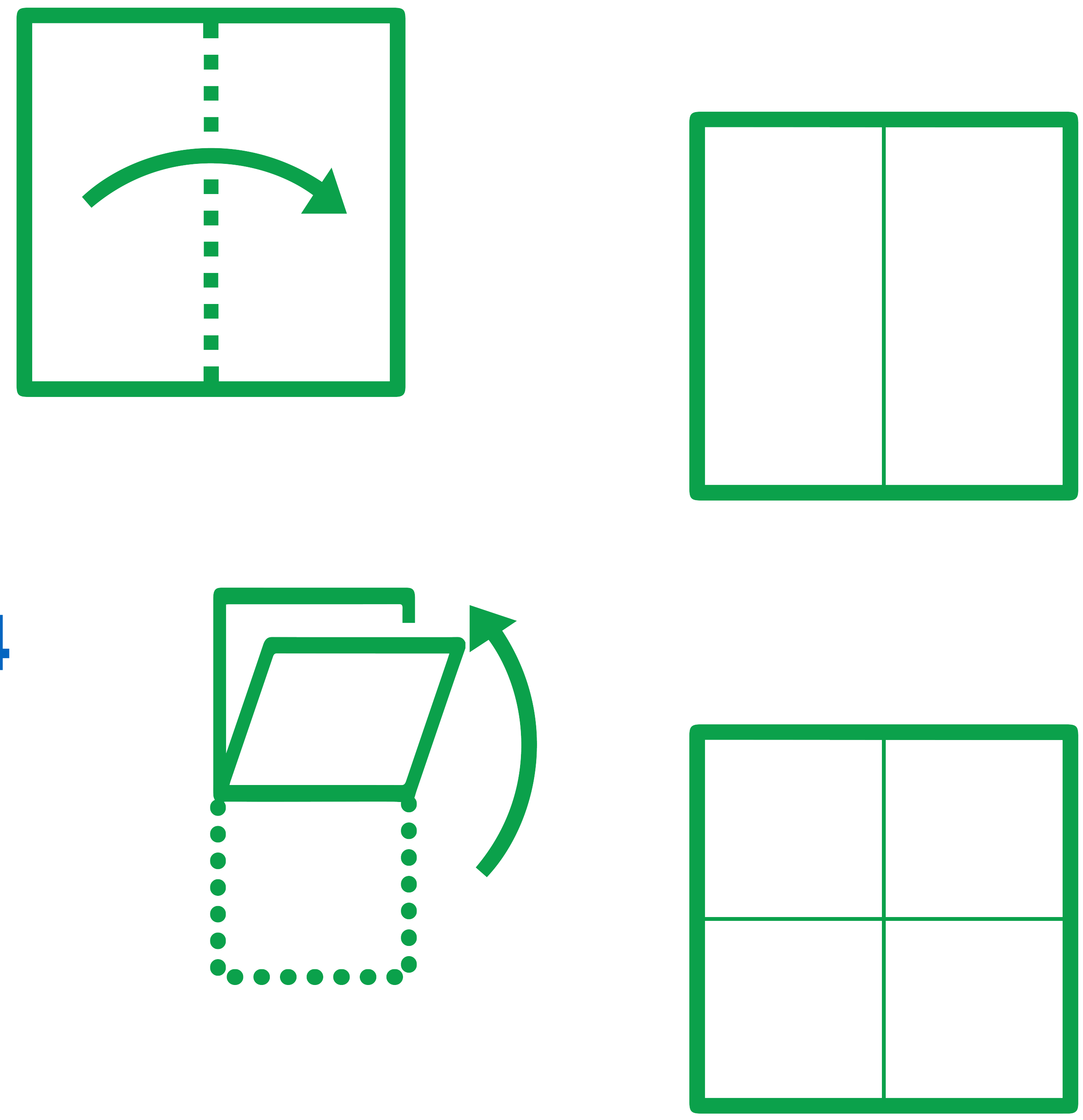
Is there a faster way?

If drawing one square is considered one step, how many steps did this algorithm take?

16

Making a Grid

1. Take a scrap of paper
2. Fold it in half.
   - We made **2** boxes!
3. Fold it in half again.
   - How many boxes now? **4**



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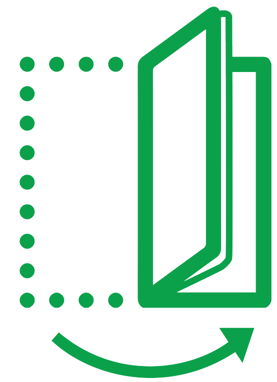
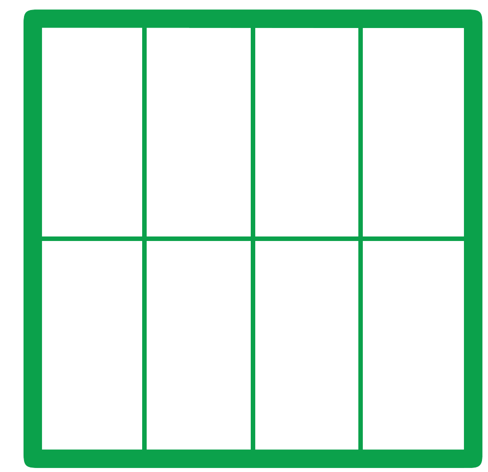
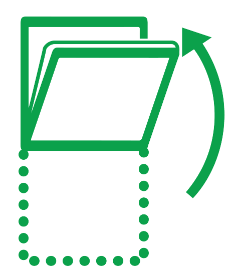
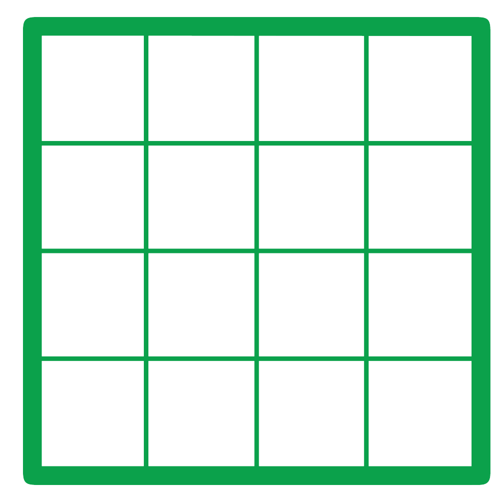




3. Fold it in half again.
   - How many boxes now? **4**
4. Fold it in half again.
   - How many boxes now? **8**





5. Fold it in half again.
   - **4** folds total
6. Open the paper. How many boxes do we have?

Making a Grid

1. Take a scrap of paper
2. Fold it in half.
   - We made **2** boxes!
3. Fold it in half again.
   - How many boxes now? **4**
4. Fold it in half again.
   - How many boxes now? **8**
5. Fold it in half again.
   - **4** folds total
6. Open the paper. How many boxes do we have? **16**

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

If folding once is considered one step, how many steps did this algorithm take to make 16 boxes?

Describe Growth of the Folding Algorithm

- 1 Fold = 2 boxes
- 2 Folds = 4 boxes
- 3 Folds = 8 boxes
- 4 Folds = 16 boxes

What is the mathematical relationship to predict the number of folds [steps] based on number of boxes [elements]?

$$f = \log_2 b$$

$$2^f = b$$

Logarithms: our favorite function

- The inverse of exponents
- $\log_{10} 100$ = ???
- How many 10s do we multiply together to get 100?
  - $\log_{10} 100 = 2$
- In computer science, "log 8" most often implies $\log_2 8$

$2^x = 16$ is equivalent to $\log_2 16 = x$

$2^4 = 16$ is equivalent to $\log_2 16 = 4$

$2^6 = 64$ is equivalent to $\log_2 64 = 6$