

Constraint Satisfaction

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May 1, 2017

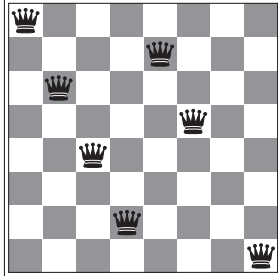
Announcements

- This week
 - Monday: Constraint satisfaction
 - Wednesday: “Mastering the Game of Go with Deep Neural Networks and Tree Search”
 - Friday: “Computing Machinery and Intelligence”
- Next week
 - Monday: Wrap-up
 - Wednesday-Friday: Final presentations and demos
- Watch for email with information about final project presentations and a survey to schedule them

Today

- Constraint Satisfaction

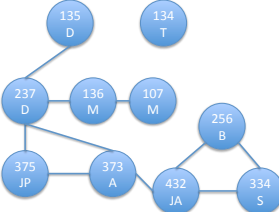
Coming full circle: back to Problem Solving



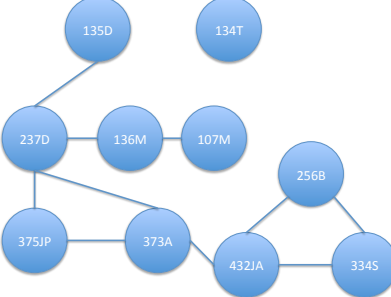
Not looking for a path. Looking for a state that satisfies a set of criteria.

A Constraint Satisfaction Problem

- Final exams are given on W, Th, F, and Sa
- No professor can give 2 exams on the same day, unless teaching two sections of the same course
- Multiple sections of 134/136/432 should be scheduled for the same day
- Students enrolled in 136 and 237 (or those enrolled in some combination of 373/375/237) shouldn't have to take those exams on the same day
- Same for students in any combination of 256/334/432
- Same for students in 373/432
- Steve must give his exam (334) on W



A Constraint Satisfaction Problem



One Solution

Wed	Thurs	Fri	Sat
334S	432JA	256B	134T
373A	237D	375JP	135D
136M	107M		

Formally

- A CSP is a triple (X, D, C) , where
- X is a set of variables
 - {107M, 134T, 135D, 136M, 237D, 256B, 334S, 373A, 375JP, 432JA}
- D is a set of domains, one for each variable
 - Domain is the same for all variables here
 - {W, Th, F, Sa}
- C is a set of constraints

Constraints for our CSP

$C = \{$
 $135D \neq 237D, 136M \neq 107M, 136M \neq 237D,$
 $373A \neq 432JA, 373A \neq 375JP \neq 237D,$
 $256B \neq 334S \neq 432JA,$
 $334S = W$
 $\}$

Another Way to Express Constraints

$C = \{$
 $\langle (135D, 237D), \{(W, Th), (W, F), (W, Sa),$
 $(Th, W), (Th, F), (Th, Sa), (F, W), (F, Th),$
 $(F, Sa), (Sa, W), (Sa, Th), (Sa, F)\} \rangle$
 $\langle (334S), \{(W)\} \rangle$
etc.
 $\}$

Global Constraints

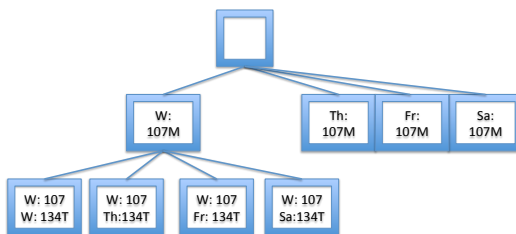
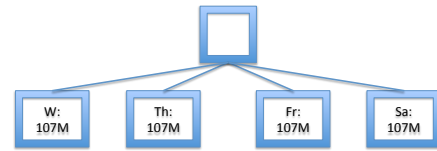
- Generally involve all variables.
- E.g., Can't schedule more than three exams on any given day.

How to Solve our CSP?

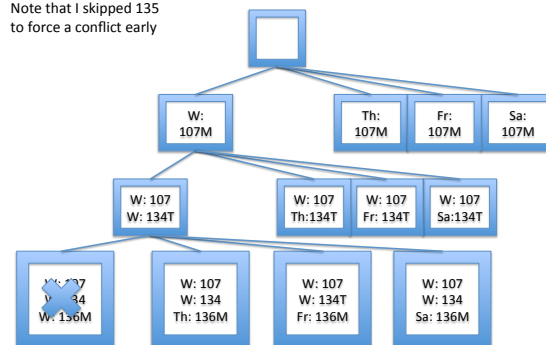
- Formulate as a search problem
- State has variables $1 \dots k$ assigned; variables $(k+1) \dots 10$ unassigned, where the order of the variables is 107M, 134T, 135D, 136M, 237D, 256B, 334S, 373A, 375JP, 434JA.
- Initial State: no variables assigned
- Goal Test: All variables assigned and no constraints violated.
- Which search method???

A Simple Improvement

- Don't ever allow a successor that is inconsistent with its parent
- Goal test is now very simple: is the variable assignment complete?
- Consistency checking + 1 var at a time: "Backtracking Search"



Note that I skipped 135 to force a conflict early



Other Simple Improvements?

Consider the order in which we assign the variables

- Might it make sense to assign 334S first?
 - **MRV (minimum remaining values) heuristic**
- Select variables involved in the largest number of constraints on other unassigned variables
 - **Degree heuristic**

Another Simple Improvement:

Forward Checking

- At start, for each variable, record the current set of possible legal values for it.
- When you assign a value in the search, update the set of legal values for all unassigned variables.
- Backtrack immediately if you empty a variable's constraint set, restoring legal values that were eliminated in the second point above.

Now Another Simple Improvement

- In what order should its values be tried?
 - Least-constraining value:** rules out fewest choices in neighbors

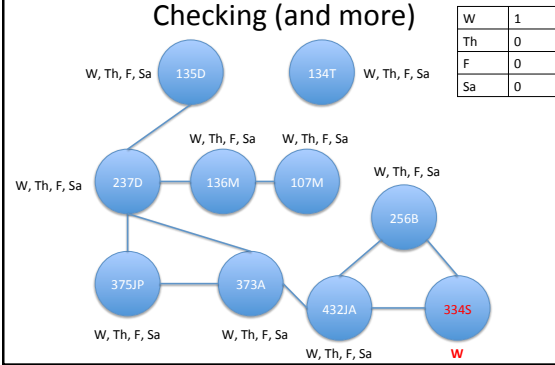
Backtracking Search

- Assign a value to one variable at a time, maintaining consistency with previous assignments
- Improvements
 - Forward Checking
 - Heuristics for "next variable" selection
 - MRV
 - Degree heuristic
 - Heuristics for "next value" assignment
 - Least constraining value

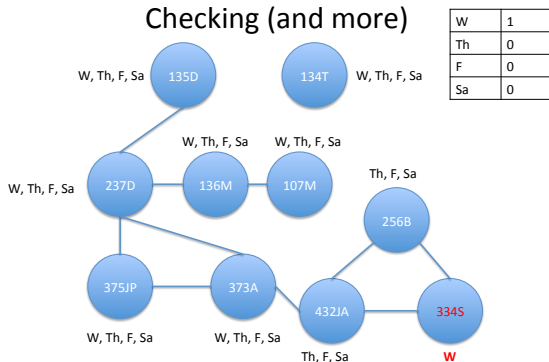
Backtracking Search

- How to backtrack?
 - Chronological backtracking**
 - If a branch of a search fails, back up to the preceding variable and try a different value

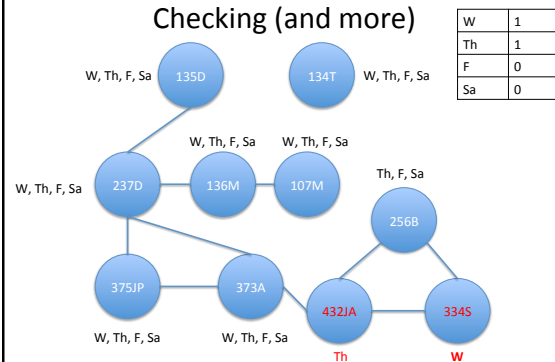
Exam Scheduling with Forward Checking (and more)

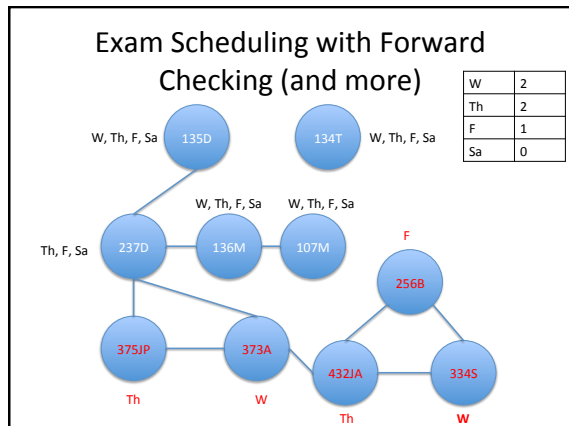
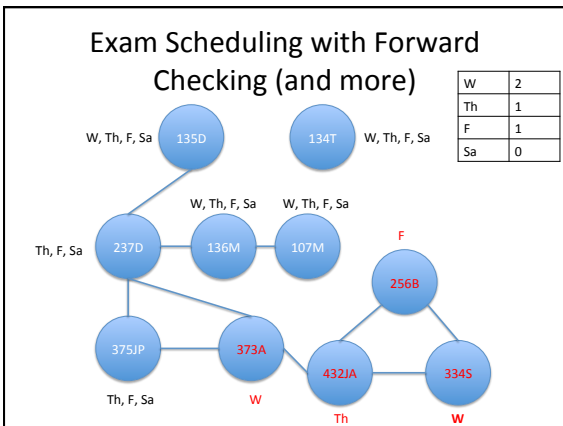
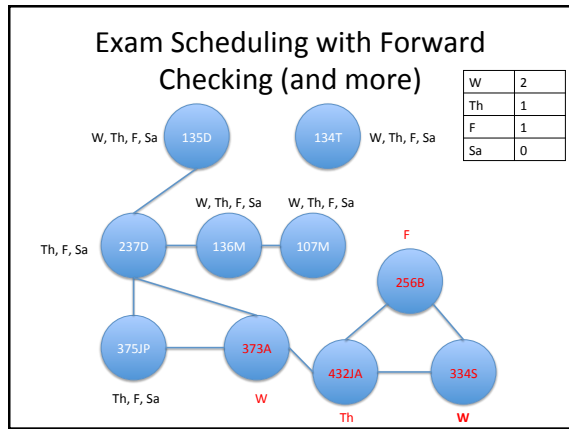
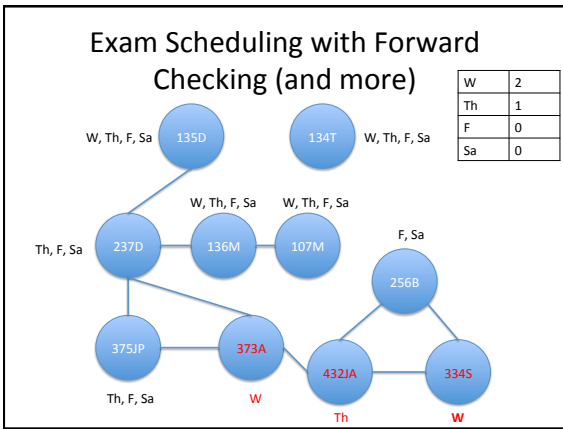
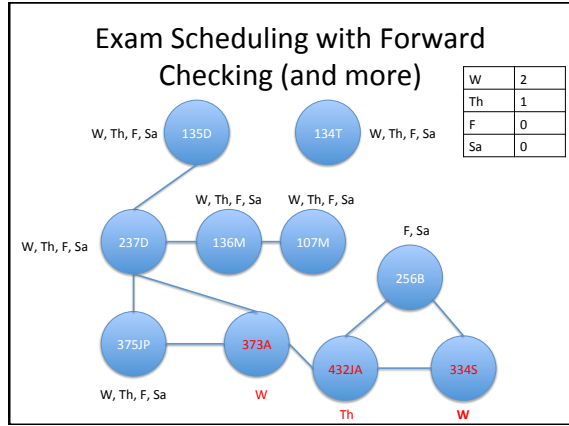
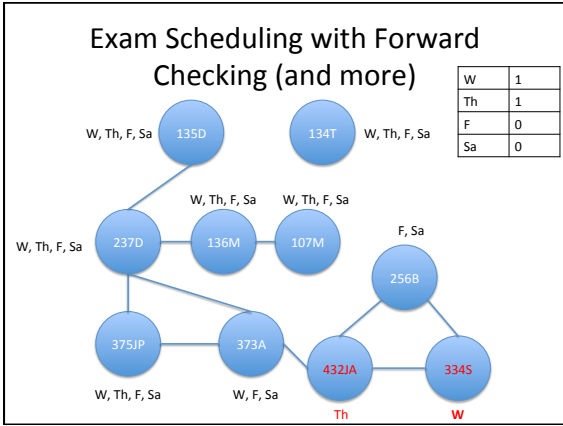


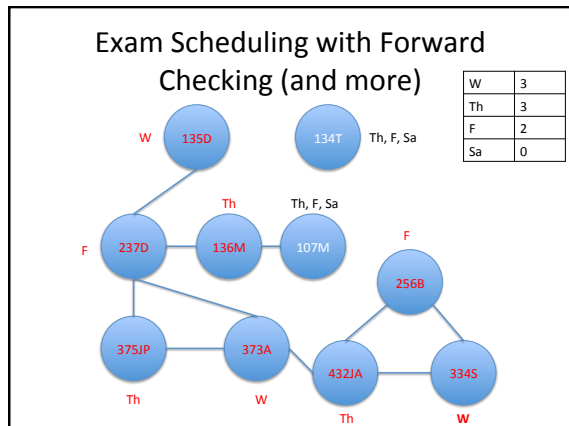
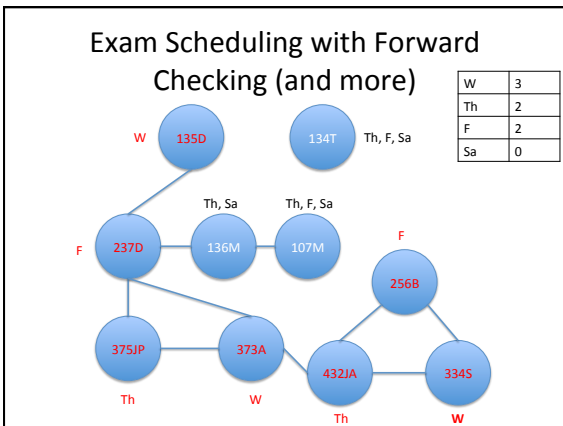
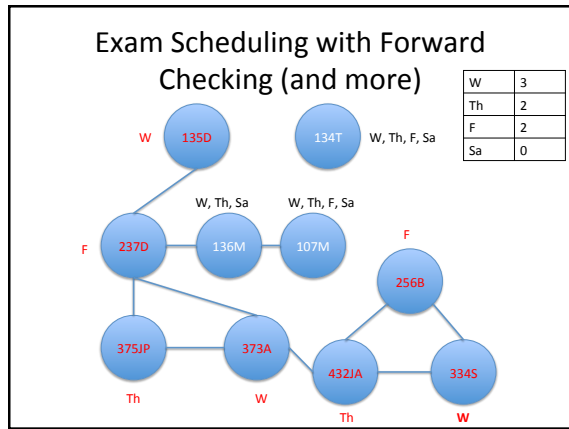
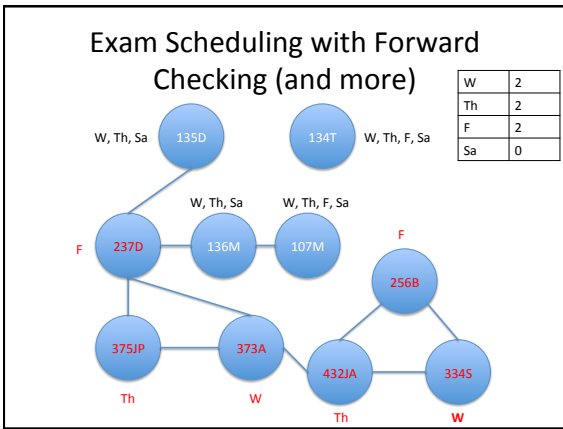
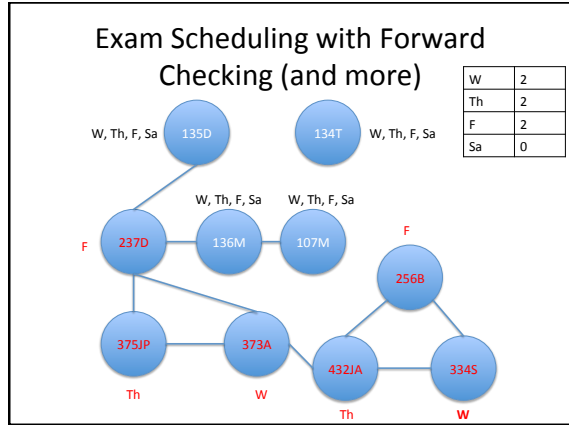
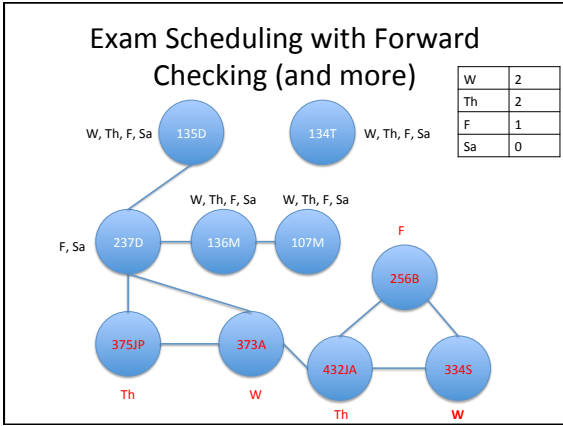
Exam Scheduling with Forward Checking (and more)

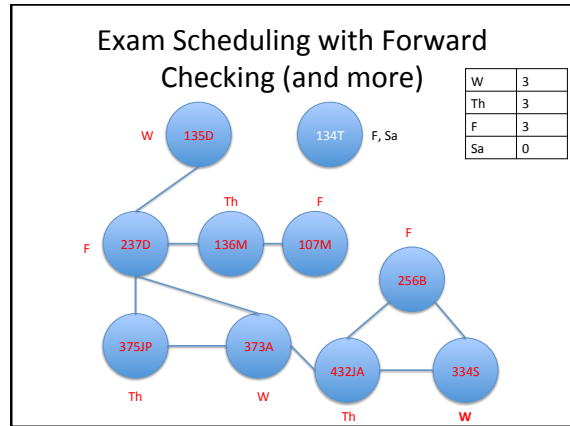
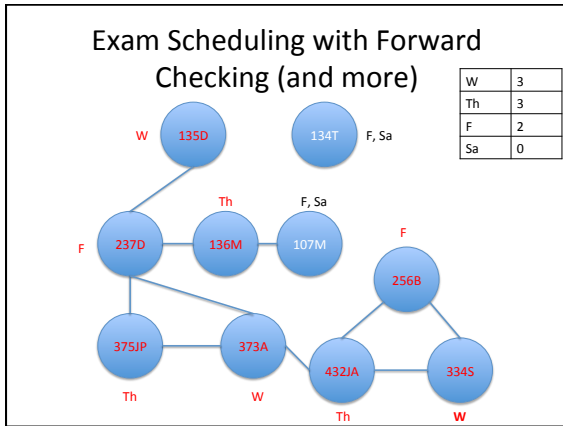


Exam Scheduling with Forward Checking (and more)









Our Solution

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135D	136M	107	

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- ### CSP Solvers vs State Space Searchers
- State Space Search
 - States are, for the most part, atomic
 - Domain-specific heuristics
 - CSP
 - Factored state representation; peek into states and consider relationships of states to each other
 - Domain-independent heuristics to speed up search

- ### Real-World Examples
- Assignment problems
 - Who teaches which class
 - Scheduling problems
 - Which exam is given when
 - Floor planning/layout

An Early AI Example Labeling Line Drawings

- Waltz's algorithm
 - Convex interior lines are labelled as +
 - Concave interior lines are labeled as -
 - Boundary lines are labeled as ->
- There are 208 intersection labelings
- But only 18 are legal:

[M. DesJardin]