Reinforcement Learning: Q-Learning

Andrea Danyluk

March 3, 2017

[Many of these slides adapted from CS 188, Stuart Russell, Andrew Moore, or Szepesvari and Sutton.]

Announcements

- Programming Assignment 2 due last night
  - Sign up for code reviews
- Assignment for Monday
  - Read Holte et al.'s AAAI 2016 paper on bi-directional search
  - Turn in brief reading response (no more than one page, 12pt font, 1.5 spacing) at start of class
- Sample midterm available online
- RL assignment now posted
  - Confirm partners with me by Monday 9am

Today's Lecture

- Reinforcement Learning
  - Finish up TD example
  - Q-Learning

The TD Algorithm

function TD(S, R, S', V)

Input: S is the current state, S' is the next state,
R is the immediate reward for the transition,
V is the array storing the current value estimates

0: if S' is a new non-terminal state, then V[S'] = R;
if terminal state, V[S'] = 0

1: $\delta \leftarrow R' + \gamma \cdot V[S'] - V[S]$

2: $V[S] \leftarrow V[S] + \alpha \cdot \delta$

3: return V

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Initialize to arbitrary values. Often we choose 0.

Example: TD Policy Evaluation

Episodes:

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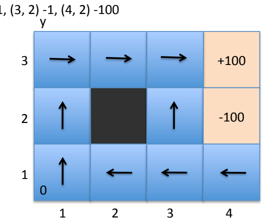

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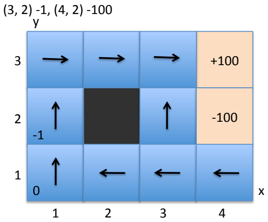

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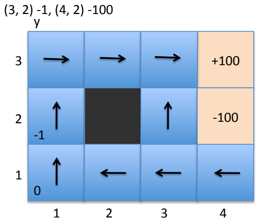

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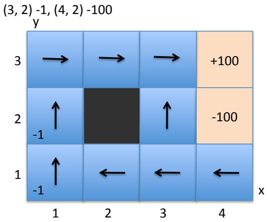

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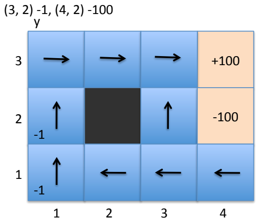

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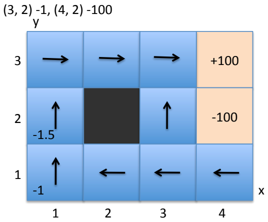

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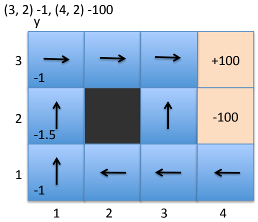

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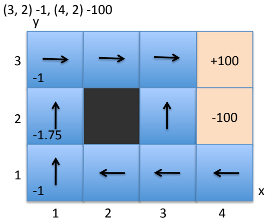

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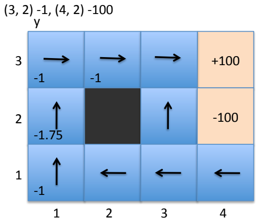

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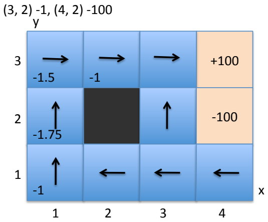

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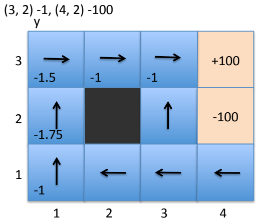

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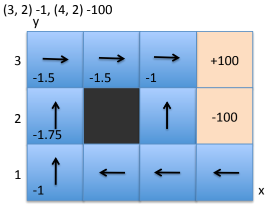

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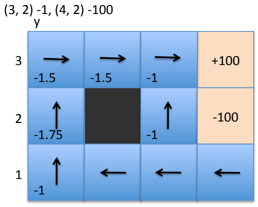

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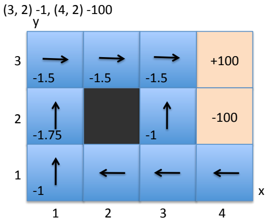

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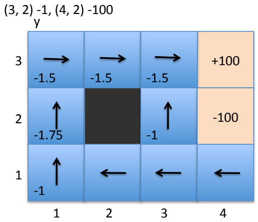

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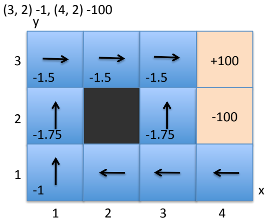

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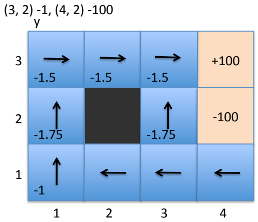

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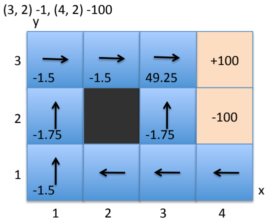

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The Problem with TD Value Learning

- We now have a model-free way to do policy evaluation
- But we can't turn it into a new policy without more work
  - Have an estimate of the state transition probabilities for the fixed policy
  - But need a model with complete probabilities for all actions
  - Can learn policy with exploring starts and generalized policy iteration

What we really want

- A model-free way to do policy evaluation (as we explore)
- Ability to move toward a new (and ultimately optimal) policy

Active RL

- Given:
  - Ability to perceive states and rewards
  - Knowledge of available actions
  - No knowledge of $P(s'|s, a)$
  - No knowledge of rewards $R(s, a, s')$
- Goal: learn **state values and optimal policy**
- Learner actively explores the world
  - Tradeoff between exploration and exploitation

Recall: Optimal Values (Utilities)

$Q^*(s, a) = \Sigma\, P(s' \mid s,a) \cdot [R(s') + \gamma \cdot V^*(s')]$,
where the sum is over all s'

$V^*(s) = \max_a Q^*(s, a)$

Aim to learn the Q values directly

Q-Learning

- An alternative TD method
- Choose an action in the given state, s. Apply that action, a. Now have:
  (s, a, s', r)
- Get sample of Q(s, a):
  sample = $R(s,a,s') + \gamma \max_{a'} Q(s', a')$
- Update Q(s, a) – i.e., compute a running average:
  $Q(s, a) = (1-\alpha)\, Q(s, a) + \alpha(\textit{sample})$

Q-Learning Properties

Q-learning converges to the optimal policy

- If the learning rate is small enough
- If you explore enough
  - Want the exploration method to be *greedy in the limit of infinite exploration*
    - Aim to try each action in each state an infinite number of times
    - Need to eventually become greedy so that the agent's actions become optimal with respect to the learned (true) model

Exploration/Exploitation Schemes

- ε-greedy selection
  - When choosing an action, flip a coin
  - With probability ε, act randomly
  - Else follow the current policy (breaking ties randomly)
- ε-greedy selection, but lower ε over time
- Give some weight to actions the agent has not tried often
- More complex selection functions

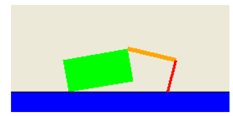
Demo 1: Mazeworld

- Magenta: taking best action
- Yellow: exploring other action
- Smiling face: successfully completes action
- Sad face: transitions to unintended state

- Remember: this demo aims to minimize value – not maximize.

Demo 2: Crawler

- States?
- Actions?
- Rewards?



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