Value and Policy Iteration

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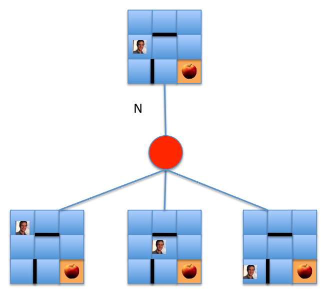
Announcements

- Programming Assignment 2 in progress
- On Wednesday will announce an article to read for Monday

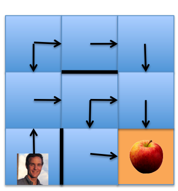
Today's Lecture

- Quick review of Value Iteration
- Policy Iteration

Stochastic Gridworld



Policies, not Plans



Value Iteration

- Will calculate successive estimates V_k^* of V^*
- Start with $V_0^*(s) = 0$ for all s
- Given V_i^*, calculate the values for all states for depth i+1

  $V_{i+1}^*(s) = \max_a \Sigma P(s' | s,a) \cdot [R(s') + \gamma \cdot V_i^*(s')]$
- Throw out old vector V_i^*
- Repeat until convergence
- Called **value update** or **Bellman update**

[Adapted from CS 188 Berkeley]

Value Iteration Demos

- All rewards are 1
- The value of a state is either the value itself or the *value + the penalty* if you got there by running into a wall (so in this case we aim to minimize expected "reward")
- PJOG = how badly you go off course
  - 0 means your action does what you intended
  - 0.3 means 70% of the time your action does what's intended; splits the 30% evenly among the remaining options
- Discount rate (gamma) is always 1

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Things to notice in the demos

- Value approximations get refined toward optimal values
- Information propagates outward from the terminal states until all states have correct information
- The policy may converge long before the values do

The Bellman Equation: a closer look

$V^*(s) = \max_a \Sigma P(s' \mid s,a) \cdot [R(s') + \gamma \cdot V(s')]$

Reconciling the formulations in the two texts:

Sutton and Barto:

$V^*(s) = \max_a \Sigma P(s' \mid s,a) \cdot [R(s,a,s') + \gamma \cdot V(s')]$

We've been taking the reward of the transition to be the reward of the state we would enter upon transition

Russell and Norvig:

$V^*(s) = R(s) + \max_a \Sigma P(s' \mid s,a) \cdot [\gamma \cdot V(s')]$

A common formulation: take the reward of the transition to be the one of the state you're in

Values (Utilities) for Fixed Policies

- How do we compute the utility of state under a fixed (not necessarily optimal) policy?

  $V^\pi(s) = \Sigma P(s' \mid s,\pi(s)) \cdot [R(s, \pi(s),s') + \gamma \cdot V^\pi(s')]$,

  where the sum is over all s'

This is the expected total discounted reward starting in s and following the policy

Policy Evaluation

- Can calculate the V's for a fixed policy just as we calculated V* earlier
- Set values to 0 initially
- Perform recursive update

  $V_{i+1}^\pi(s) = \Sigma P(s' \mid s,\pi(s)) \cdot [R(s, \pi(s),s') + \gamma \cdot V_i^\pi(s')]$,

  where the sum is over all s'

Note: No "max" here. So this is just a set of linear equations that can be solved without recursive update.

Policy Iteration

Repeat

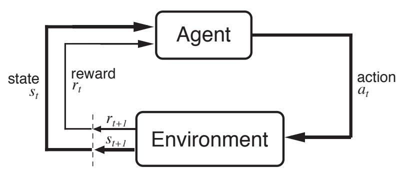
- Step 1: Policy evaluation
  - Calculate utilities for fixed (probably suboptimal) policy until convergence (in practice, a reasonable approximation is good enough)
- Step 2: Policy improvement
  - Update policy using one-step lookahead

Until policy converges

Reinforcement Learning

- Assume an MDP
  - S: a set of states
  - A: a set of actions
  - P(s’ | s, a): the probability of ending up in state s’, given that the agent is in state s and takes action a
  - R(s): or R(s, a, s’): a reward function
  - Want to find a policy π
- But this time we don’t know P or R
  - Need to try things out in order to learn

Reinforcement Learning



- Will assume agent can observe the state it’s in.
- Agent receives feedback in the form of rewards.
- Learns to act so as to maximize expected return.

Passive RL

- Given:
  - A policy π(s)
  - No knowledge of P(s’|s, a)
  - No knowledge of rewards R(s, a, s’)
- Goal: learn state values (not policy yet…)
  - Recall policy evaluation!
- Passive in the sense that there’s no choice about what actions to take
  - Need to *execute* the policy to learn from experience
  - Not offline planning. Actually *take actions* to learn.

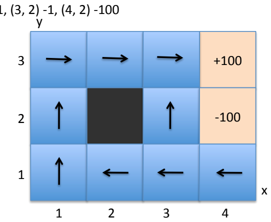
Example: Direct Estimation

Episodes:

(1, 1) -1 , (1, 2) -1 , (1, 2) -1 , (1, 3) -1, (2, 3) -1, (3, 3) -1, (3, 2) -1, (3, 3) -1, (4, 3) +100

(1, 1) -1, (1, 2) -1, (1, 3) -1, (2, 3) -1, (3, 3) -1, (3, 2) -1, (4, 2) -100

$V(s) = E\,[\Sigma\, \gamma^t R(S_{t+1})]$,
t from 0 to ∞
$s = S_0$



γ = 1, R = -1

V(2, 3) = (96 + -103) / 2 = -3.5

V(3, 3) = (99 + 97 + -102) / 3 = 31.3