

Lecture 9

Homework #9: 2.3.1, 2.3.2, 2.3.3, 2.3.5, 2.3.6 a & f, 2.3.10

For 2.3.3, give both the construction and the proof of correctness.
[Note that this might become Homework #10, depending on how much we can or can't cover today.]

Thm. For each NFA, there is an equivalent DFA.

Proof continued from last class...

$$(q,w) \vdash_{M^1}^* (p,e) \text{ iff } (E(q),w) \vdash_{M^1}^* (P,e), \quad p \in P.$$

Induction Step. Assume that the claim is true for all w , $|w| \leq k$, $k \geq 0$.

Show that it also holds for w , $|w| = k + 1$.

Let $w = va$, $v \in \Sigma^*$, $a \in \Sigma$.

$$\begin{aligned} (\Rightarrow) \text{ Suppose that } (q,w) \vdash_{M^1}^* (p,e), \text{ which we can rewrite as} \\ (q,va) \vdash_{M^1}^* (p,e) \\ = (q, va) \vdash_{M^1}^* (r^1, a) \vdash_{M^1} (r^2, e) \vdash_{M^1}^* (p,e) \end{aligned}$$

Now, what can we say about the pieces of this computation?

First, rather than considering $(q, va) \vdash_{M^1}^* (r^1, a)$, let's think about $(q, v) \vdash_{M^1}^* (r^1, e)$. The induction hypothesis tells us that

$$(E(q),v) \vdash_{M^1}^* (R^1,e), r^1 \in R^1.$$

Second, since $(r^1, a) \vdash_{M^1} (r^2, e)$
 (r^1, a, r^2) is in Δ^1
so $E(r^2) \subseteq \delta''(R^1, a)$, by definition.

Third, since $(r^2, e) \vdash_{M^1}^* (p,e)$, $p \in E(r^2)$.

Putting these all together:

$$p \in E(r^2) \subseteq \delta''(R^1, a), \text{ so } (R^1,a) \vdash (P,e), \quad p \in P.$$

So

$$(E(q), va) \vdash^*_M (R^1, a) \vdash (P, e), \quad p \in P.$$

so

$$(E(q), va) \vdash^*_M (P, e), \quad p \in P.$$

The other direction is easier - we won't do it in class.

The proof technique followed above (construction, or simulation, as the text refers to it), will be a valuable tool:

We'll often want to take finite automata and construct new finite automata from them. Among other things, this will allow us to show that regular languages can be combined in various ways and still remain regular. (don't forget, though we haven't yet shown it, the languages accepted by finite automata are the regular languages!)

[Note: we'll also be showing that various combinations of languages are not regular, but that's an entirely different story. . .]

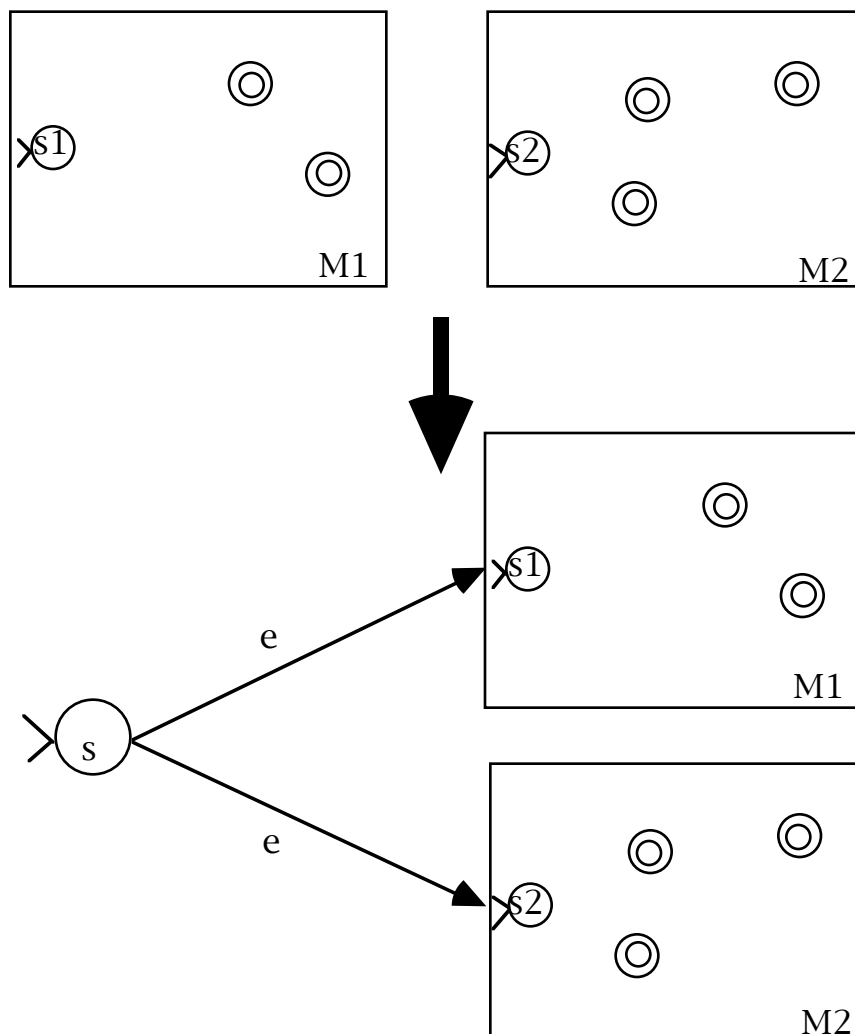
Thm. The class of languages accepted by finite automata is closed under

- union
- concatenation
- Kleene star
- complementation
- intersection

Union

Let L_1 and L_2 be languages accepted by M_1 and M_2 , respectively (where M_1 and M_2 are NFA).
i.e., $L_1 = L(M_1)$ and $L_2 = L(M_2)$.

Let's "look at" the NFAs first:



Let $M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$
and $M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$

assume that K_1 and K_2 are disjoint (if not, you can rename the states of one without affecting its operation)

assume that both use the same alphabet, though the new finite automaton can be designed for the alphabet that is the union of Σ_1 and Σ_2 , if not.

Let a new NFA $M = (K, \Sigma, \Delta, s, F)$, where

$$K = K_1 \cup K_2 \cup \{s\}$$

$$F = F_1 \cup F_2$$

$$\Delta = \Delta_1 \cup \Delta_2 \cup \{(s, e, s_1), (s, e, s_2)\}$$

if $w \in \Sigma^*$, then

$$(s, w) \xrightarrow{*}_M (q, e), q \in F$$

iff

$$(s_1, w) \xrightarrow{*}_{M^1} (q, e), q \in F_1 \text{ or}$$

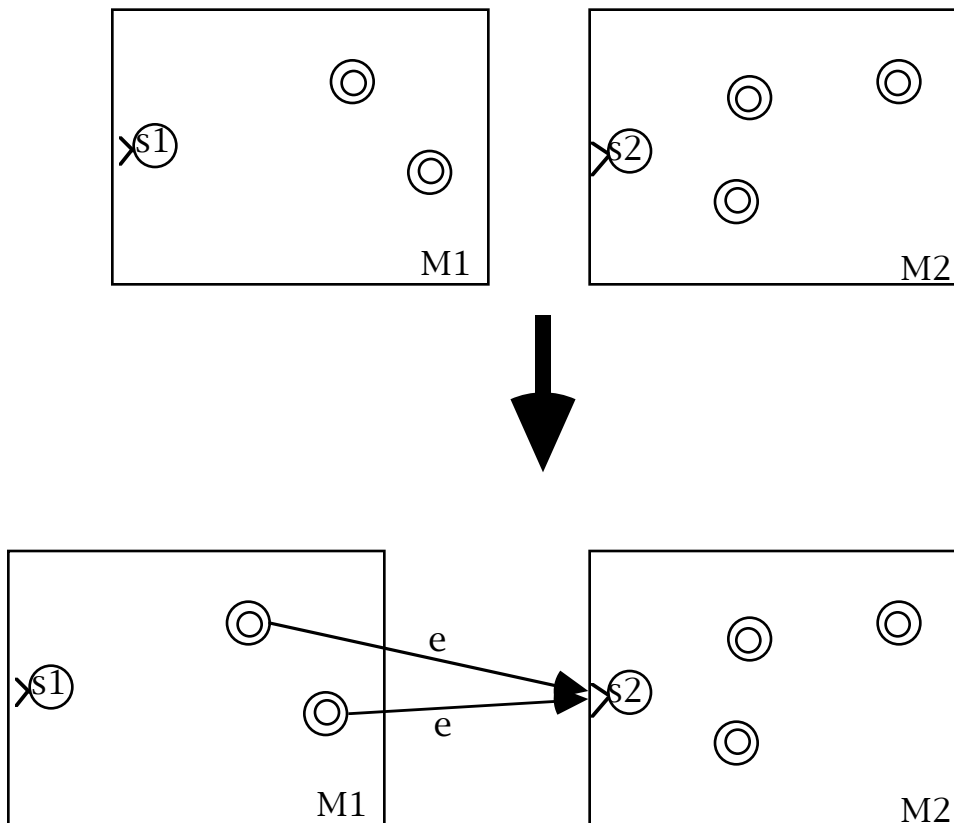
$$(s_2, w) \xrightarrow{*}_{M^2} (q, e), q \in F_2.$$

Note: since these constructions are obvious, we won't prove their correctness, though we are normally required to do so.

Concatenation

Let L_1 and L_2 be languages accepted by M_1 and M_2 , respectively (where M_1 and M_2 are NFA).
i.e., $L_1 = L(M_1)$ and $L_2 = L(M_2)$.

Let's "look at" the NFAs again:



Let $M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1)$
and $M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$

assume that K_1 and K_2 are disjoint (if not, you can rename the states of one without affecting its operation)

assume that both use the same alphabet, though, as in the previous case, this is not necessary.

Let a new NFA $M = (K, \Sigma, \Delta, s, F)$, where

$$K = K_1 \cup K_2$$

$$s = s_1$$

$$F = F_2$$

$$\Delta = \Delta_1 \cup \Delta_2 \cup \{(f, e, s_2) : f \in F_1\}$$

if $w \in \Sigma^*$, then

$$(s, w) \xrightarrow{*}_M (q, e), q \in F$$

iff

$$(s_1, u) \xrightarrow{*}_{M^1} (q_1, e), q_1 \in F_1 \text{ and}$$

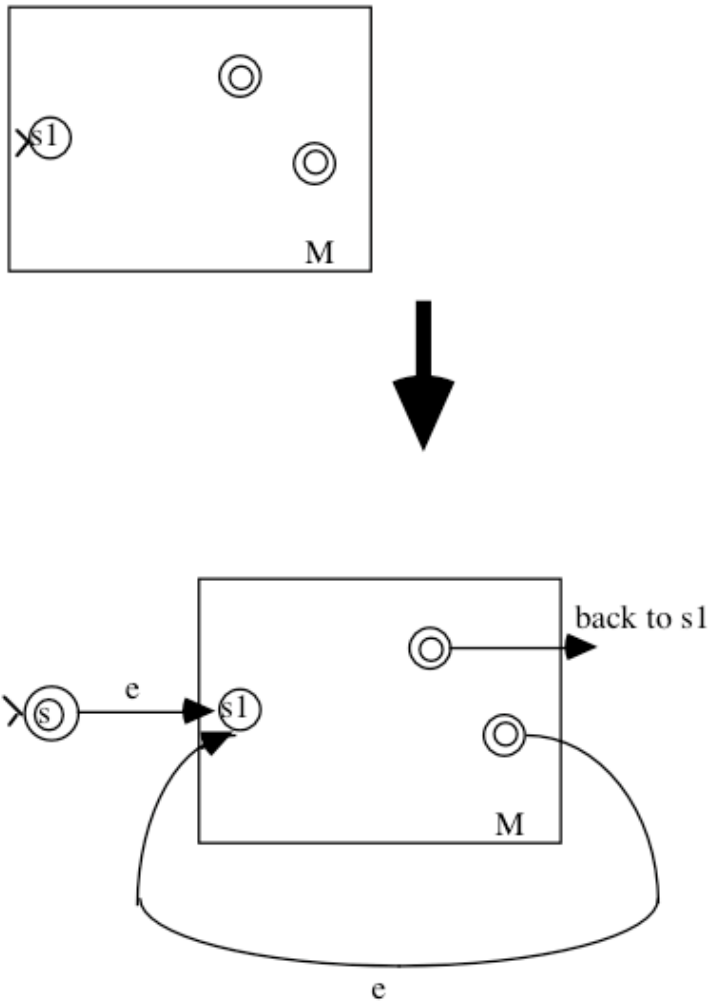
$$(s_2, v) \xrightarrow{*}_{M^2} (q_2, e), q_2 \in F_2 \text{ and}$$

$$w = uv, u, v \in \Sigma^*$$

Kleene Star

Let L be the language accepted by M , where M is an NFA.
So $L = L(M)$.

Let's "look at" the NFA once more:



Let $M = (K, \Sigma, \Delta, s_1, F)$

Let a new NFA $M' = (K', \Sigma, \Delta', s, F')$, where

$$K' = K \cup \{s\}$$

$$F' = F \cup \{s\}$$

$$\Delta' = \Delta \cup \{(s, e, s1)\} \cup \{(f, e, s1) : f \in F\}$$

if $w \in \Sigma^*$, then

$$w = e \text{ or}$$

$$(s, w) \xrightarrow{*M'} (q, e), q \in F'$$

iff

$$w = w_1 w_2 w_3 \dots w_n, \text{ each } w_i \in \Sigma^*$$

and

$$(s1, w1) \xrightarrow{*M'} (q1, e), q1 \in F \text{ and}$$

$$(s1, w2) \xrightarrow{*M'} (q2, e), q2 \in F \text{ and}$$

\dots

$$(s1, wn) \xrightarrow{*M'} (q, e), q \in F$$

Complementation ($\Sigma^* - L(M1)$)

$$\text{Let } M1 = (K1, \Sigma, \delta1, s1, F1)$$

NOTE: A DFA!!!

$$\text{Let } M = (K1, \Sigma, \delta1, s, K1 - F1)$$

Basically, we're just flipping the final and non-final states.

We've assumed that $M1$ is a DFA. What if it were an NFA? Would this still work?

Intersection

Note that $L1 \cap L2 = \Sigma^* - (\Sigma^* - L1) \cup (\Sigma^* - L2)$

