Lecture 4

Homework #4: 1.6.1 c, d, e, 1.6.2, 1.6.5, 1.7.2 c, 1.7.3
[For 1.7.2c, the book says to use the inductive definition of reversal. You might find this useful, but you might find other things more useful.]

Finishing up proof techniques:

The Diagonalization Principle

Example. Show that [0,1) is uncountable.

Assume the contrary, i.e., that [0,1) is countable. Then the elements can be itemized as follows:

$x_1 = 0.\ a_{11}\ \ a_{12}\ \ a_{13}\ \ \ldots$
$x_2 = 0.\ a_{21}\ \ a_{22}\ \ a_{23}\ \ \ldots$
$x_3 = 0.\ a_{31}\ \ a_{32}\ \ a_{33}\ \ \ldots$

etc.

Where each x_i is the decimal expansion of a number between 0 and 1.

Now corrupt each digit along the diagonal - i.e.,

let d_{11} be some $n \in \{1, .., 8\} \neq a_{11}$
let d_{22} be some $n \in \{1, ..., 8\}$ a $\neq$ $_{22}$
etc.

but then $0.\ d_{11}d_{22}\ldots$ isn't enumerated!

Closure:

Let D be a set, and let $n \geq 0$,
Let $R \subseteq D^{n+1}$, an (n+1)-ary relation on D.
A subset B of D is closed under R if
$b_{n+1} \in B$ whenever $b_1, \ldots, b_n \in B$
in $(b_1, \ldots, b_n, b_{n+1}) \in R$.

odd int closed under multiplication? YES.
positive integers closed under subtraction? NO.

R* is the reflexive, transitive closure of a binary relation $R \subseteq A \times A$
$= R \cup \{(a,b)$: there is a path in R from a to b$\}$

it is the closure of R under reflexivity and transitivity.

Now, finally moving on to the topic of this course:

Since we'll be investigating models of computing, we need a way to represent data: strings

Definitions:

An <u>alphabet</u> is a finite set of symbols.
ex. {0,1}

A <u>string (word)</u> over an alphabet is a finite sequence of symbols.
ex. 00111, 0, abc, aaabbb

a special string is the <u>empty string</u> = e

The set of all strings over an alphabet ∑ is denoted ∑*.

The <u>length</u> of a word is its length as a sequence
ex. $|e| = 0$

The <u>concatenation</u> of two strings x and y is denoted xy or x°y.
(xy is a new sequence comprised of the sequence x followed by the sequence y)

a special case is x°e = e°x

A string v is a <u>substring</u> of w if $|v| \leq |w|$ and w = xvy for some x, y. (x and/or y can be empty)

The <u>reversal</u> of a string is the string written backward.

Let w ∈∑ *. The reversal of w, denoted w^R is as follows:

<u>Def (inductive)</u>:

1. if w is a string of length 0, then
   $w^R = w = e$.
2. if w is a string of length (n+1) > 0,
   then w = ua for some a ∈∑ and
   $w^R = au^R$

NOTE:

in genl, will use a, b, c to denote symbols;
x, y, z, u, v, w to denote words

can use the def above to show that $(uv)^R = v^R u^R$

Examples: of inductive definitions and proofs for strings.

I. a palindrome is a string w such that $w = w^R$

Inductive Def:
e is a palindrome
a is a palindrome for each $a \in \Sigma$

if $a \in \Sigma$ and x is a string and a palindrome, then
axa is a palindrome.

II. Show that $(w^R)^R = w$, for any string w.

Will show by induction on the length of w.

Basis: Let w = e. Then $(w^R)^R = (e^R)^R = e^R = e = w$.

I.H. Assume $(w^R)^R = w$ for $|w| \leq n$.

Induction Step. Let $|w| = n+1$. Then w = ua for some $u \in \Sigma^*$ and $a \in \Sigma$ such that $|u| = n$.

$(w^R)^R = ((ua)^R)^R = (a^R u^R)^R = (u^R)^R (a^R)^R = ua = w$.