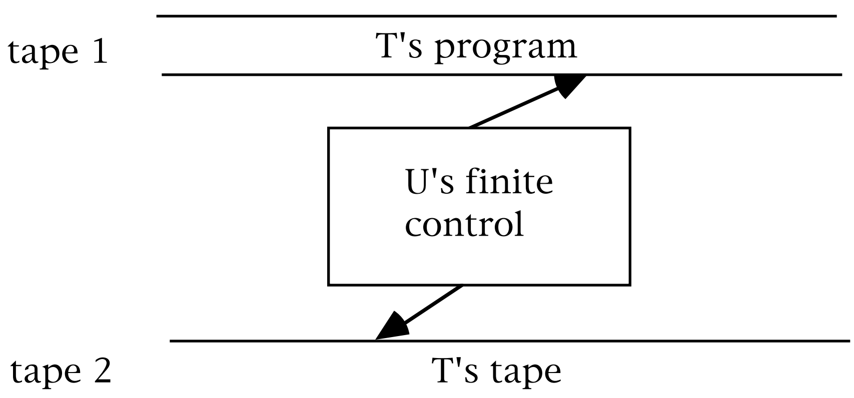
Lecture 34

By definition any given Turing Machine runs a fixed program. We have discussed the fact that the program might be an interpreter than runs other programs. Today we see how that works.

Universal Turing Machine:

- has a fixed program permanently embedded in its finite control
- the fixed program can mimic the action of an arbitrary TM (i.e., it's an interpreter)
- program is written on one tape and mimicked on another (so there are two tapes)



The fixed program in U is really just an interpreter:

(1) Given: T's current state and input symbol
Find: quintuple in description of T that applies

(q, s, q', s', d), where
q = current state
s = current symbol
q' = next state
s' = symbol to write
d = direction to move

*Note extension to standard TM. Why is this ok?

(2) Record: state q'
Simulate on tape 2: write s'; move tape 2's read/write head

Record: new symbol from tape 2

Universal machine U expects a particular format for the description of T's program:

will use binary alphabet to represent

- quintuples of T
- T's tape symbols

(1) How many bits are needed for each quintuple?

n states: need $\lceil \log n \rceil$ bits to encode

let $k = \lceil \log n \rceil$

2 tape moves (L and R): 1 more bit to encode

each tape square: 1 bit

$\Rightarrow$ 2k + 3 bits needed to encode each quintuple

(2) How to separate quintuples?

$\Rightarrow$ use special symbol X

(3) Boundary markers for program?

$\Rightarrow$ special symbol Y

U's interpreter program:

(only part of it, actually)

<u>Phase I</u>

locate next quintuple to be executed - i.e., find q & s

where will q and s be stored?

- could use 3rd tape -or-
- use (k+1)-bit segment immediately to left of the left Y-marker

Algorithm.

- start with read/write head on left Y-marker
- scan to left

  change 0 to A

  change 1 to B

  until blank
- scan to right

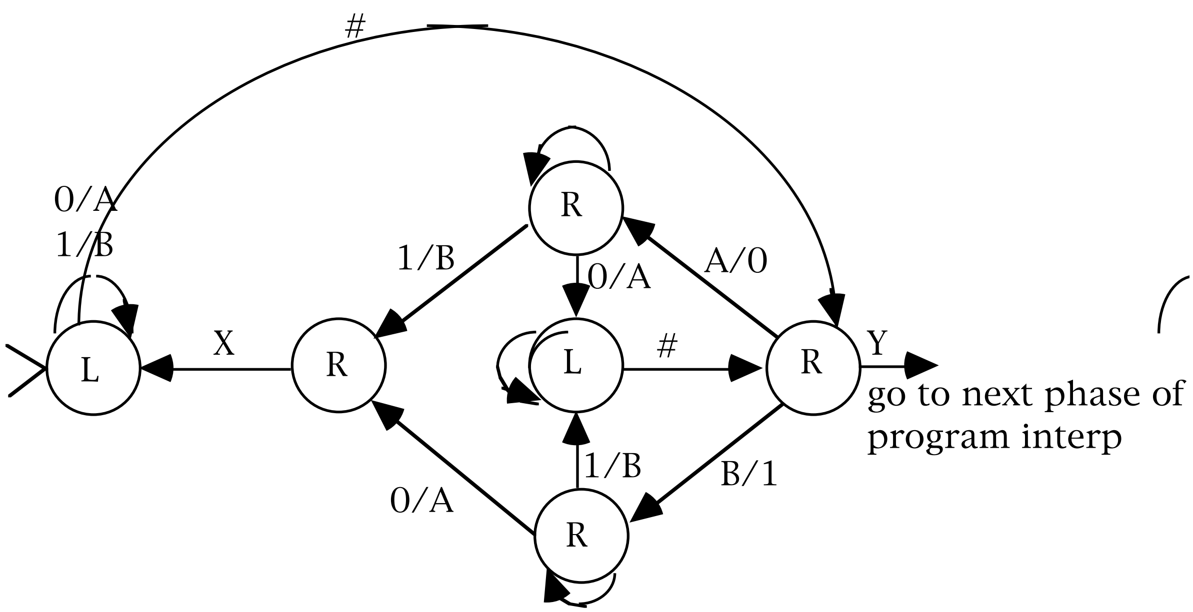
  if A, change to 0

  else if B, change to 1

  move right and search for match

  if you've moved past an X, need to start this cycle again (i.e., you've moved to the next quintuple without having matched)

- to guarantee skipping over a non-matching quintuple, change 0's to A and 1's to B.



*Note difference in TM representation

Phase II

Algorithm.

- record new state q' in workspace
- move head on tape 2
- record new symbol

at start:

q and s from quintuple are A's and B's;

but

q' and s' and d are still binary

r/w head on left-hand Y-marker

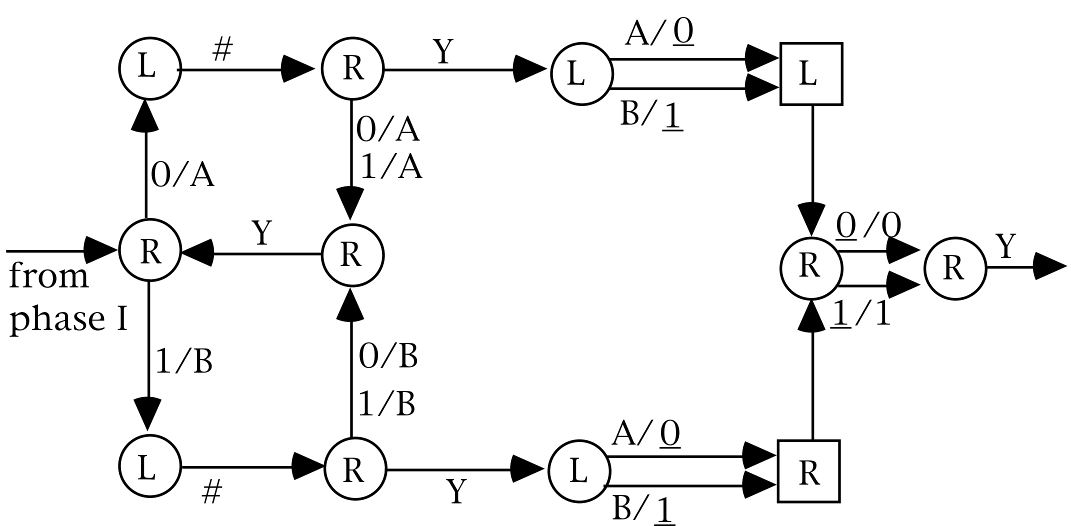
Algorithm at next level of detail.

- scan right to binary squares
- copy to workspace (in terms of A's and B's)
- encounter s'; copy to workspace
- scan right and pick up d; no more room in workspace; so "remember" d
- pick up s'
  convert to 0/1 and write on tape 2
- move tape 2's r/w head
- read next s from tape 2

In the following TM

squares = tape 2

underlines = tape 2



Now -- how do things get started?

- r/w head #1 over left-hand Y-marker
- r/w head #2 over initial cell of tape 2
- initialize workspace