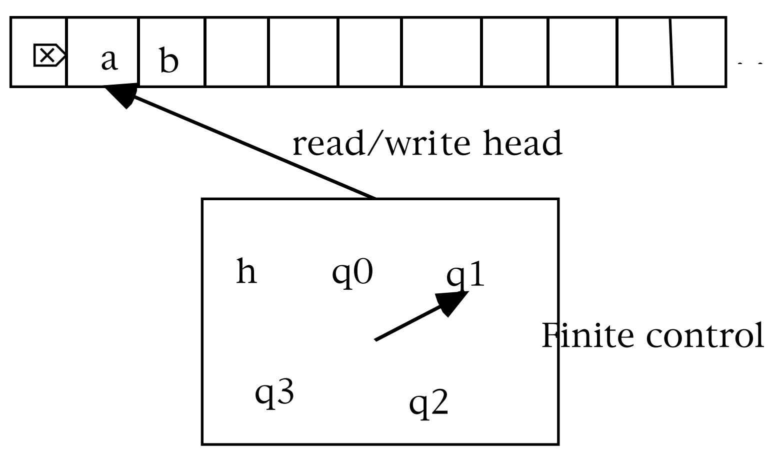
Lecture 24

Homework #24: 4.1.1 - 4.1.5, 4.1.7 (hand in 4.1.4, 4.1.7)

Note that there are many typos in Chapter 4 in my copy of the text. Corrected in yours?

And now finally....

Today we begin to talk about yet another model of computation: the Turing Machine. The most interesting property of the Turing Machine is that there is no known model that is more powerful. That is, we will argue that any way of formalizing the idea of an algorithm is equivalent to the idea of a Turing Machine. But this will happen much later. First, we'll spend some time defining and understanding Turing Machines.



Operation of a Turing machine:
Depending upon the current state and the tape symbol currently scanned:

1. Set the finite control to a new state.
2. Either
   write a symbol in the tape square currently being scanned or
   move the read/write head 1 square to the left or to the right.

Note that the tape is infinite to the right.

If the machine attempts to move off the left end of the tape, it runs into the left end marker, >, which pushes the r/w head one square to the right.

Input is placed at the left end of the tape.
Special **halt states** signal that a computation is over. These are not necessarily like final states. They simply indicate that computation has stopped - but say nothing about the outcome of the computation.

Notation:

blank symbol will be denoted #; (note that the symbol in the text is different)

movement of the tape head will be denoted ← and → for left and right, respectively.

Formally:

A Turing Machine (TM) is a quintuple (K, Σ, δ, s, H), where

K is a finite set of states;
Σ is an alphabet, containing # and >, but not ← and →;
$s \in K$ is the initial state;
$H \subseteq K$ is the set of halt states;
δ is a function from (K-H) x Σ to (K) x (Σ ∪ {←, →}) such that,

(a) for all $q \in K\text{-}H$, if $\delta(q, >)=(p,b)$, then $b= \rightarrow$.
(b) For all $q \in K\text{-}H$ and $a \in \Sigma$, if $\delta(q,a)=(p,b)$ then $b \neq >$.

if $\delta(q, a) = (p, b)$, then M, when in state q and scanning a, enters state p, and either writes b, if b is a symbol, or moves 1 square left or right, if b is ← or →.

operation is deterministic - only stops when M enters a halting state.

Example 1. Changes all a's to b's.

M = (K, Σ, δ, s, H), where
$K = \{q_0, h\}$
$\Sigma = \{a, b, \#, >\}$
$s = q_0$
$H = \{h\}$

δ given by the following table:

q	σ	$\delta(q,\sigma)$
q_0	a	(q_0, b)
q_0	b	$(q_0, \rightarrow)$
q_0	#	$(h, \#)$
q_0	>	$(q_0, \rightarrow)$

*Unless told otherwise, assume that the tape head is placed initially on the first tape square immediately to the right of ⌦

Example 2. Adds 2 pos integers in unary.

$M = (K, \Sigma, \delta, s, H)$, where

$K = \{q_0, q_1, q_2, h\}$

$\Sigma = \{I, \#, >\}$

$s = q_0$

$H = \{h\}$

δ given by the following table:

q	σ	$\delta(q,\sigma)$	
q_0	I	$(q_0, \rightarrow)$	
q_0	#	(q_1, I)	
q_1	I	$(q_1, \rightarrow)$	
q_1	#	$(q_2, \leftarrow)$	
q_2	I	$(h, \#)$	
q_2	#	$(h, \#)$	[note that this won't happen]

all transitions on > go to $(q_0, \rightarrow)$

Note that while we define a set of halting states, the TM need not enter a halting state on all input. It might run indefinitely.

Now, (as always) we need to formalize the notion of computation and discuss terminology:

A **configuration** of a TM $M = (K, \Sigma, \delta, s, H)$ is a member of:

$K \times {>}\Sigma^* \times (\Sigma^*(\Sigma - \{\#\}) \cup \{e\})$

We represent the string to the left of the tape head, including the symbol scanned by the tape head, and the string to the right of the tape head.

for right of tape head, list only "significant" symbols.

We'll write (q, wa, u) as $(q, w\underline{a}u)$ **abbreviated notation.**

A **halted configuration** is one in which the state is a member of H.

The relation **yields in one step** between configurations is defined as follows:

Let $M = (K, \Sigma, \delta, s, H)$ be a TM and
let $(q_1, w_1\underline{a}_1u_1)$ and $(q_2, w_2\underline{a}_2u_2)$ be configurations of M.

Then

$$(q_1, w_1\underline{a}_1u_1) \vdash (q_2, w_2\underline{a}_2u_2)$$

iff, for some $b \in \Sigma \cup \{\leftarrow, \rightarrow\}$, $\delta(q_1, a_1) = (q_2, b)$ and either

1. $b \in \Sigma$, $w_1 = w_2$, $u_1 = u_2$, and $a_2 = b$, or

2. $b = \leftarrow$, $w_1 = w_2a_2$, and either
   (a) $u_2 = a_1u_1$, if $a_1 \neq \#$ or $u_1 \neq e$, or
   (b) $u_2 = e$, if $a_1 = \#$ and $u_1 = e$, or

3. $b = \rightarrow$, $w_2 = w_1a_1$, and either
   (a) $u_1 = a_2u_2$ or
   (b) $u_1 = u_2 = e$ and $a_2 = \#$.

Other terms: reflexive transitive closure of $\vdash$.
computation.
length of a computation.