Lecture 19

Homework #20: 3.5.1, 3.5.3 a
Hand in: 3.5.1 a, b; 3.5.3 a

Just as we found that many languages are not regular, we will also see that many languages are not context free.

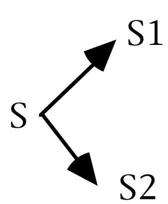
In the next few classes we'll see how to establish that a language is context free . . . or not.

Thm. The context free languages are closed under union, concatenation, and Kleene star.

Let $G_1 = (V_1, \Sigma_1, R_1, S_1)$
$G_2 = (V_2, \Sigma_2, R_2, S_2)$

Union. Construct G such that $L(G) = L(G_1) \cup L(G_2)$

Let S be a new start symbol.



G = $(V_1 \cup V_2 \cup \{S\},$
$\Sigma_1 \cup \Sigma_2,$
$R1 \cup R2 \cup \{S \to S_1, S \to S_2\},$
$S)$

Concatenation. Construct G such that $L(G) = L(G_1)L(G_2)$

Let S be a new start symbol.

$$G = (V_1 \cup V_2 \cup \{S\},\ \Sigma_1 \cup \Sigma_2,\ R1 \cup R2 \cup \{S \rightarrow S_1S_2\},\ S)$$

<u>Kleene Star.</u> Construct G such that L(G) = L(G1)*

First, let's consider how we get a*

$A \rightarrow aA$
$A \rightarrow e$

Here we'll apply a similar idea:

$S \rightarrow S_1S$
$S \rightarrow e$

$$G = (V_1 \cup \{S\},\ \Sigma_1,\ R1 \cup \{S \rightarrow S_1S, S \rightarrow e\},\ S)$$

Some examples that make use of the closure results:

$\{a^m b^n c^k : m, n, k \geq 0, m = n \text{ or } n = k \text{ or } m = k\}$

This can be expressed as:

$\{a^m b^n c^k : m, n, k \geq 0, m = n\} \cup$
$\{a^m b^n c^k : m, n, k \geq 0, n = k\} \cup$
$\{a^m b^n c^k : m, n, k \geq 0, m = k\}$

Another example.

$\{a^m b^m a^n b^n: m, n \geq 0\}$

Can be expressed as:

$\{a^m b^m: m \geq 0\} \{a^n b^n: n \geq 0\}$

Aside: $a^m b^m c^m$ is not context-free.

Thm. The intersection of a context-free language with a regular language is a context-free language.

Pf. Here the idea is similar to something we did for regular languages. We will create a new machine that will simulate two machines in parallel.

Let $L = L(M_1)$ $M_1 = (K_1, \Sigma_1, \Gamma_1, \Delta_1, s_1, F_1)$ a PDA

Let $R = L(M_2)$ $M_2 = (K_2, \Sigma_2, \delta_2, s_2, F_2)$ a DFA

Construct $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$K = K_1 \times K_2$

$\Sigma = \Sigma_1 \cup \Sigma_2$

$\Gamma = \Gamma_1$

$s = (s_1, s_2)$

$\Delta = \{(((q_1, q_2), u, \beta)), ((p_1, p_2), \gamma))$

iff

$(((q_1, u, \beta), (p_1, \gamma)) \in \Delta_1$

and $(q_2, u) \vdash^* (p_2, e)\}$