Lecture 16

Homework #16: 3.2.2, 3.2.3, 3.2.4b

Today: a different way to represent derivations.

Consider

$S \to AB$

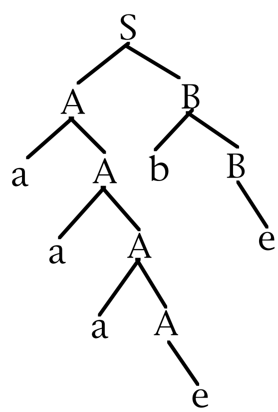
$A \to aA$ $\quad A \to e$ $\quad B \to bB$ $\quad B \to e$

and now consider a new representation of a derivation:

Parse Tree:

root = genly the start symbol
intermediate nodes = non-terminals
leaves = terminals

read the terminals left to right.

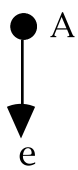


string is: aaab

Def. Parse tree, root, leaves, yield for an arbitrary $G = (V, \Sigma, R, S)$.

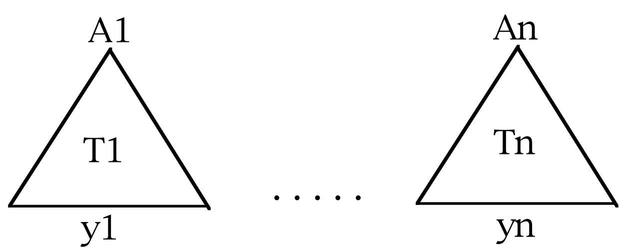
(1) •a is a parse tree.
the single node is both a root and a leaf.
Note: $a \in \Sigma$.

(2) if A → e is a rule in R



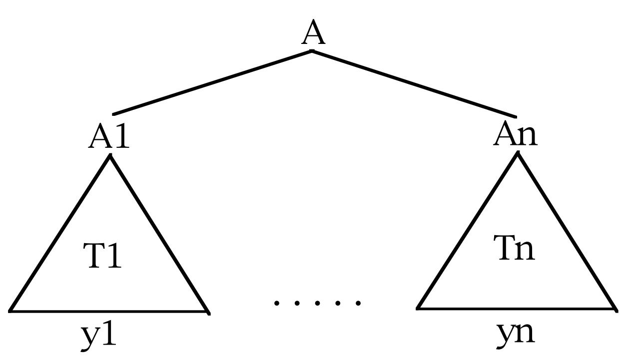
is a parse tree
the root is labeled A
the one leaf is labeled e
yield is e.

(3) if



are parse trees and A → A1 . . . An ∈ R

then



is a parse tree.
the root is A
the leaves are the leaves of the constituent trees

yield is y1...yn

Def. A path is a sequence of nodes from root to leaf.

The height of the parse tree is the length of the longest path.

Parse trees represent derivations of strings in L(G) so that the superficial differences between derivations, owing to the **order** of application of rules, are suppressed.

Leftmost and rightmost derivations

A leftmost derivation exists in every parse tree – obtained by repeatedly replacing the leftmost non-terminal.

A rightmost derivation exists in every parse tree – obtained by repeatedly replacing the rightmost non-terminal.

Thm. Let G = (V, Σ, R, S) be a context-free grammar, and let $A \in V-\Sigma$, and $w \in \Sigma^*$. Then the following stmts are equivalent:

(a) $A \Rightarrow^* w$
(b) There is a parse tree with root A and yield w.
(c) There is a leftmost derivation $A \Rightarrow^{L*} w$.
(d) There is a rightmost derivation $A \Rightarrow^{R*} w$.

Again, parse trees represent derivations without superficial differences of order of rule application. However...

Parse trees will be different, of course, for a different **choice** of rules to apply.

For instance, consider the following grammar for generating boolean expressions without parentheses.

R: E → E || E   E → E && E
E → ! E   E → B   (Boolean value)
E → N   (Named Boolean var)
B → true | false
N → x1 | x2 | x3

E ⇒ E || E ⇒ ! E || E ⇒ ! N || E ⇒ ! x1 || E ⇒ ! x1 || N

$\Rightarrow$! x1 || x2

E $\Rightarrow$ E || E $\Rightarrow$ E || N $\Rightarrow$! E || N $\Rightarrow$! N || N $\Rightarrow$! x1 || N
$\Rightarrow$! x1 || x2

vs

E $\Rightarrow$! E $\Rightarrow$! E || E $\Rightarrow$! N || E $\Rightarrow$! x1 || E $\Rightarrow$! x1 || N
$\Rightarrow$! x1 || x2

The first two generate the same parse tree. The third is completely different.

Grammars with strings that have two or more distinct parse trees are called **ambiguous**.

Can sometimes disambiguate a grammar by restructuring it.

There are some CFLs with the property that all CFGs generating them are ambiguous – call these inherently ambiguous.

Ex. $\{a^i b^j c^k \mid i=j \text{ or } i=k\}$

Every string $a^n b^n c^n$ has two distinct derivations.