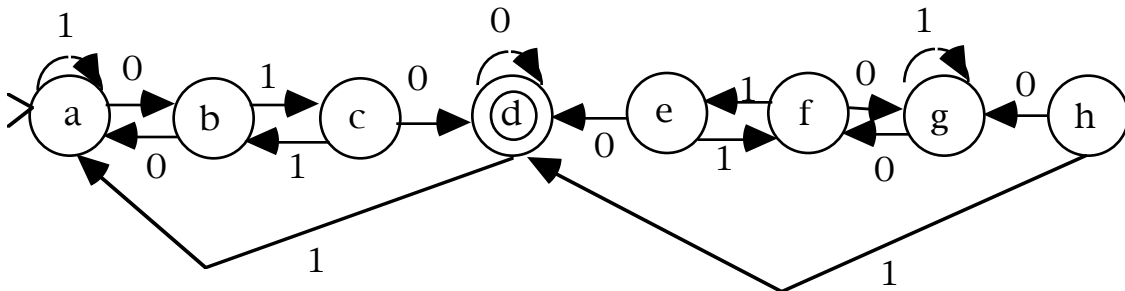


Lecture 13

Homework #13: Find a minimum state DFA equivalent to:



Go through the process, even though the solution is obvious!!! It's designed to be obvious so that you can check your solution easily. Remember to show all work for full credit.

Thm. The minimum state automaton accepting L is unique up to an isomorphism (i.e., a renaming of the states).

Let's look at an algorithm for finding a min state DFA, M^1 , equivalent to a given

$$M = (K, \Sigma, \delta, s, F)$$

Let \equiv be an equivalence relation on the states of M .

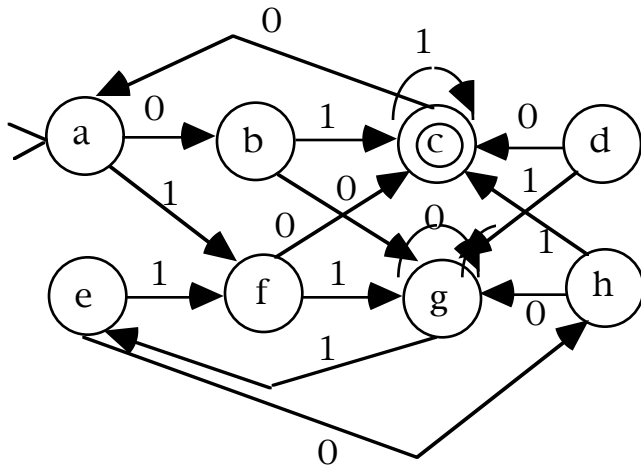
$$p \equiv q \text{ iff}$$

for input x ,

$$(p, x) \vdash^* (f_1, e) \text{ iff } (q, x) \vdash^* (f_2, e), f_1, f_2 \in F.$$

Algorithm is based on finding equivalent states (or, actually, on finding pairs of states that are "distinguishable" - i.e., not equivalent)

Let's develop the algorithm while tracing through an example:



We begin by creating a table, where entries correspond to pairs of states.

b	X						
c	X	X					
d	X	X	X				
e		X	X	X			
f	X	X	X		X		
g	X	X	X	X	X	X	
h	X		X	X	X	X	X
	a	b	c	d	e	f	g

We write an "X" when a pair is not equivalent.

Start by marking all pairs where:
 One state is an accept state and
 the other is not.

[here, every pair with c in it]

Now, let's work "backward".

Look at pairs and consider where they end up after looking at one symbol.

(r,s): if $\delta(r,0)$ = final state
and $\delta(s,0)$ = non-final
then $r \neq s$, so mark it.

Do the same for all symbols in the alphabet.

so now we'll mark:

(a,b)	(b,d)	(d,e)	(e,f)	(f,g)	(g,h)
(a,d)	(b,e)	(d,g)	(e,h)	(f,h)	
(a,f)	(b,f)	(d,h)			
(a,h)	(b,g)				

What we're doing more generally is the following:

Look at pairs of states p, q that we don't know to be distinguishable.

$$r = \delta(p,a) \quad s = \delta(q,a)$$

Look at r and s, the states reached from p and q.

If r and s are marked "X", then mark p and q as well.

So now look at

(a,e) \rightarrow (b,h) on 0; f on 1.

(a,g) \rightarrow ~~(b,g) on 0.~~

(b,h) \rightarrow g on 0, c on 1.

(d,f) \rightarrow c on 0, g on 1.

(e,g) \rightarrow ~~(g,h) on 0.~~

a \equiv e,

b \equiv h,

f \equiv d

The min state DFA is:

